*APPENDIX A:** MODEL DEVELOPMENT FOR EXAMPLE 1

Value to Air Force as Function of Aptitude and Job Difficulty

The policy of Example 1 will start with a polynomial of degree 1 in A (Aptitude) and degree 3 in D (Difficulty). When policy-specifying uses a polynomial form, great simplification is obtained by starting with a model in which the variables are expressed as deviations from a well-chosen constant. The constant is chosen according to the properties specified by the policy.

The starting model is

$$
\begin{aligned}
Y \quad = \quad & B(0,0) + B(0,1) * (D - DK) + B(0,2) * (D - DK) ** 2 + B(0,3) * (D - DK) ** 3 + \\
& B(1,0) * (A - AK) + B(1,1) * (A - AK) * (D - DK) \\
& \qquad + B(1,2) * (A - AK) * (D - DK) ** 2 \\
& \qquad + B(1,3) * (A - AK) * (D - DK) ** 3
\end{aligned}
$$

Since the policy seemed to imply that D-slopes would equal zero at D = 40 for all values of A, it was decided to choose DK = 40. Since A was only of degree equal one, the value of AK could be set to either of the critical values (AK = 40 or AK = 95) discussed in the specifying process. AK = 95 was the selected value. The model now becomes

$$
\begin{aligned}
Y \quad = \quad & B(0,0) + B(0,1) * (D - 40) + B(0,2) * (D - 40) ** 2 + B(0,3) * (D - 40) ** 3 + \\
& B(1,0) * (A - 95) + B(1,1) * (A - 95) * (D - 40) \\
& \qquad + B(1,2) * (A - 95) * (D - 40) ** 2 \\
& \qquad + B(1,3) * (A - 95) * (D - 40) ** 3
\end{aligned}
$$

There are eight unknown parameters, B's, to be determined from policy specifications.

Statement 1, Restrictions 1 - 2

D-slopes (i.e., slope of Y with respect to D) are equal zero at D = 40 for all values of A.

Letting Y1D = the first partial derivative of Y with respect to D

$$
\begin{aligned}
Y1D \quad = \quad & B(0,1) + 2 * B(0,2) * (D - 40) + 3 * B(0,3) * (D - 40) ** 2 + B(1,1) * (A - 95) + 2 \\
& * B(1,2) * (A - 95) * (D - 40) + 3 * B(1,3) * (A - 95) * (D - 40) ** 2
\end{aligned}
$$

At D = 40

$$Y1D = B(0,1) + B(1,1) * (A - 95)$$

Then setting

$$Y1D = 0 = B(0,1) + B(1,1) * (A - 95)$$

we observe that for the above statement to be true for all values of A it is necessary that

$$B(0,1) = 0 \quad \text{(Restriction 1)}$$
$$B(1,1) = 0 \quad \text{(Restriction 2)}$$

Then the model can be reduced to the six parameter form to simplify imposing the remaining restrictions.

---

*All capital letters are used consistently throughout the Appendixes. Note that * means "multiplication" and ** means "exponentiation."

$$Y = B(0,0) + B(0,2) * (D - 40) ** 2 + B(0,3) * (D - 40) ** 3$$
$$+ B(1,0) * (A - 95) + B(1,2) * (A - 95) * (D - 40) ** 2$$
$$+ B(1,3) * (A - 95) * (D - 40) ** 3$$

and

$$Y1D = 2 * B(0,2) * (D - 40) + 3 * B(0,3) * (D - 40) ** 2$$
$$+ 2 * B(1,2) * (A - 95) * (D - 40) + 3 * B(1,3) * (A - 95) * (D - 40) ** 2$$

Statement 2, Restriction 3

D-slope equal zero at D = 100, A = 95

$Y1D = 0 = 2 * B(0,2) * (100 - 40) + 3 * B(0,3) * (100 - 40) ** 2$

$0 = 2 * B(0,2) * 60 + 3 * B(0,3) * (60) ** 2$

$B(0,2) + 90 * B(0,3) = 0$ **(Restriction 3)**

Statement 3, Restriction 4

The inflection point (second partial derivative of Y with respect to D = 0) occurs at A = 40, D = 40.

The second partial derivative of Y with respect to D is

$$Y2D = 2 * B(0,2) + 6 * B(0,3) * (D - 40)$$
$$+ 2 * B(1,2) * (A - 95) + 6 * B(1,3) * (A - 95) * (D - 40)$$

Evaluating at A = 40, D = 40 and setting Y2D = 0

$2 * B(0,2) + 2 * B(1,2) * (40 - 95) = 0$

$B(0,2) + (-55) * B(1,2) = 0$ **(Restriction 4)**

Statements 4-7, Restrictions 5-8

The values of Y were specified for four different combinations of A and D. The value of Y = −250 at A = 40, D = 100 was determined by experiment to make positive Y values begin just above the eligibility cut-off scores.

Then the requirements are

Y = 100 at A = 95 D = 100
Y = 35 at A = 95 D = 40
Y = 15 at A = 40 D = 40
Y = −250 at A = 40 D = 100

Evaluating the function at the above values we obtain

$B(0,0) + B(0,2) * (60) ** 2 + B(0,3) * (60) ** 3 = 100$ **(Restriction 5)**

$B(0,0) = 35$ **(Restriction 6)**

$B(0,0) + B(1,0) * (-55) = 15$ **(Restriction 7)**

$$B(0,0) + B(0,2) * (60) ** 2 + B(0,3) * (60) ** 3$$
$$+ B(1,0) * (-55) + B(1,2) * (-55) * (60) ** 2$$
$$+ B(1,3) * (-55) * (60) ** 3 = -250$$ **(Restriction 8)**

Restrictions 3-8 can now be summarized as follows:

	B(0,0)	B(0,2)	B(0,3)	B(1,0)	B(1,2)	B(1,3)	
R-3		1	90				= 0
R-4		1			−55		= 0
R-5	1	$(60)^2$	$(60)^3$				= 100
R-6	1						= 35
R-7	1			−55			= 15
R-8	1	$(60)^2$	$(60)^3$	−55	$(-55)(60)^2$	$(-55)(60)^3$	= −250

These six restrictions can be imposed as follows:

From R-6

B(0,0) = 35

From R-7

35 - 55 * B(1,0) = 15 and B(1,0) = .3636

From R-3 and R-5

(R-3) B(0,2) + 90 * B(0,3) = 0

(R-5) B(0,0) + B(0,2) * (60) * * 2 + B(0,3) * (60) * * 3 = 100

Substituting B(0,0) = 35 in (R-5) and multiplying (R-3) by (60) * * 2

(R-5) B(0,2) * (60) * * 2 + B(0,3) * (60) * * 3 = 65

(R-3) B(0,2) * (60) * * 2 + B(0,3) * 90 * (60) * * 2 = 0

B(0,3) = 65/((60) * * 2) * (−30)

B(0,3) = (−13)/(21600) = −.0006019

Substituting in R-3

(R-3) B(0,2) + 90 * B(0,3) = 0

B(0,2) = ((−90) * (−13))/(21600) = 65/1200

B(0,2) = 13/240 = .05417

Substituting in R-4

(R-4) B(0,2) + (−55) * B(1,2) = 0

B(1,2) = (−13)/(240 * (−55))

B(1,2) = .0009848

Substituting in R-8

B(1,3) = (−135)/(−55 * (60) * * 3)

B(1,3) = 1/88000 = .00001136

Then the policy-specified model of Example 1 is

Y = 35 + .05417 * (D - 40) * * 2 - .0006019 * (D - 40) * * 3
+ .3636 * (A - 95) + .0009848 * (A -95) * (D - 40) * * 2
+ .00001136 * (A - 95) * (D - 40) * * 3

APPENDIX B: MODEL DEVELOPMENT FOR EXAMPLE 2

Value to Air Force as Function of Time Used and Fraction of Fill

The policy of Example 2 starts with a polynomial of degree 1 for T (Time Used) and for F (Fraction of Fill).

The general model is

$$Y = B(0,0) + B(0,1) * (F - FK) + B(1,0) * (T - TK) + B(1,1) * (T - TK) * (F - FK)$$

Since the policy desires specific Y values when F = 0 and when T = 0, it is convenient to set FK = 0 and TK = 0. Then the model becomes

$$Y = B(0,0) + B(0,1) * F + B(1,0) * T + B(1,1) * T * F$$

There are only four unknown parameters to be specified; therefore, the four critical Y values are sufficient to determine the unknown B's.

Statements 1-4, Restrictions 1-4

Y = 100	T = 180	F = 0
Y = 0	T = 0	F = 1.0
Y = K	T = 0	F = 0
Y = K	T = 180	F = 1.0

These statements are then used to obtain the restrictions as follows:

R-1	B(0,0) + B(1,0) * 180	= 100
R-2	B(0,0) + B(0,1)	= 0
R-3	B(0,0)	= K
R-4	B(0,0) + B(0,1) + B(1,0) * 180 + B(1,1) * 180	= K

Then we have from R-3

$$B(0,0) = K$$

from R-2

$$B(0,1) = -K$$

from R-1

$$B(1,0) = (100 - K)/180$$

from R-4

$$B(1,1) = (2 * K - 100)/180$$

Then the model becomes

$$Y = K + (-K) * F + ((100 - K)/180) * T + ((2 * K - 100)/180) * T * F$$

and when K = 25

$$Y = 25 + (-25 * F) + (.4167 * T) + (-.2778 * T * F)$$

and when K = 75

$$Y = 75 + (-75 * F) + (.1389 * T) + (.2778 * T * F)$$

Value to Air Force as Function of Fill and Known Goal

This model can be developed from observing the sketch in Figure C-1 of the policy statements for three goals.

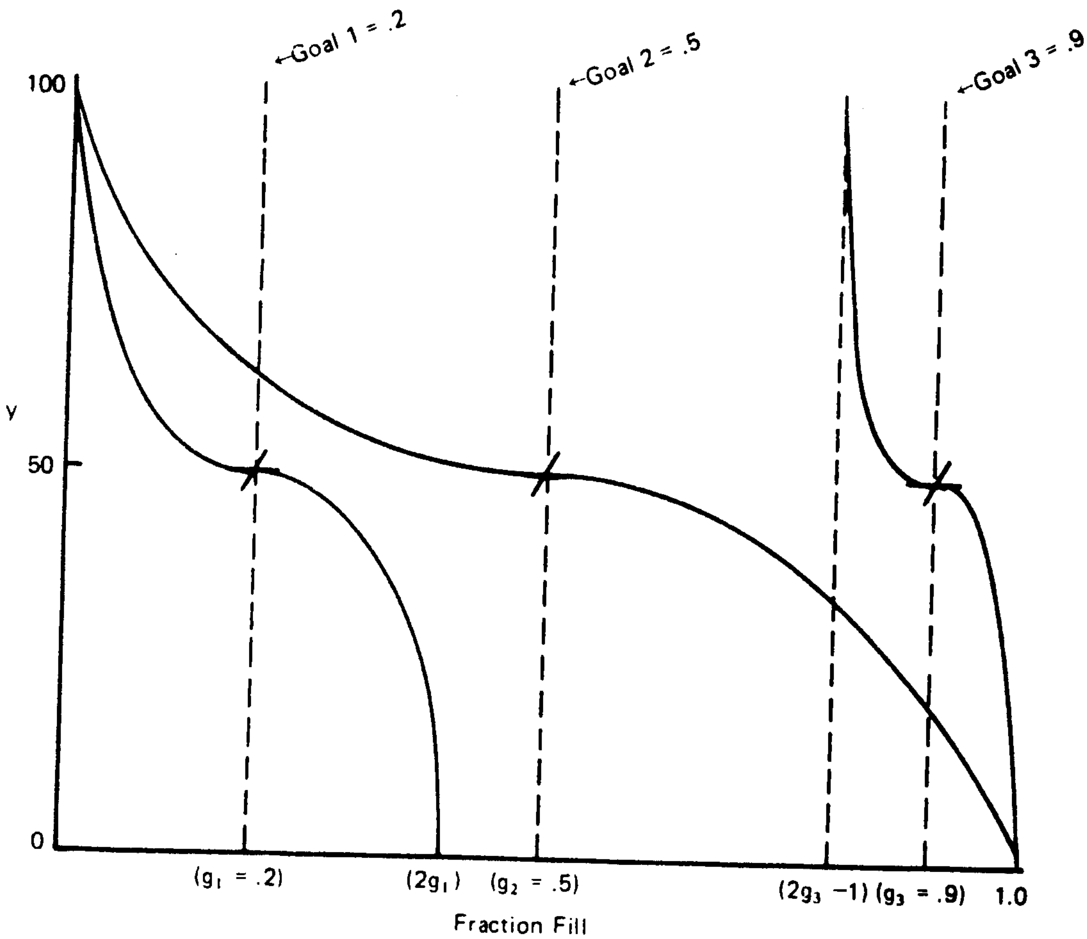

Figure C-1 Sketch of policy from example 3 showing the function for three different goals.

The model starts with a general polynomial of degree 3 for variable F.

$$Y = B(0) + B(1) * (F - FK) + B(2) * (F - FK) ** 2 + B(3) * (F - FK) ** 3$$

Since the critical values of Y are specified when F = the known goal (G), we will let the value of FK = G. Then our model is

$$Y = B(0) + B(1) * (F - G) + B(2) * (F - G) ** 2 + B(3) * (F - G) ** 3$$

There are four parameters to be determined at this time.

Statements 1 and 2, Restrictions 1 and 2

We would like the slope of Y with respect to F to be zero when F = G and the inflection point to occur when F = G.

Therefore we write the first derivative

$$Y1F = B(1) + 2 * B(2) * (F - G) + 3 * B(3) * (F - G) ** 2$$

and the second derivative

$$Y2F = 2 * B(2) + 6 * B(3) * (F - G)$$

Then we have the restrictions that

Y1F = 0 at F = G

Y2F = 0 at F = G

therefore

B(1) = 0 = Y1F

B(2) = 0 = Y2F

Then our first two restrictions imply B(1) = 0 and B(2) = 0 so the model becomes

$$Y = B(0) + B(3) * (F - G) ** 3$$

Statements 3 and 4, Restrictions 3 and 4

First consider $0 < G \leqslant .5$

Then whenever F = G, we want Y = 50 and when F = 2G, Y = 0.

Then we impose these two restrictions and obtain:

B(0) = 50

B(0) + B(3) * (G) ** 3 = 0

B(3) = −50/G ** 3

Then we have the model for $0 < G \leqslant .5$

$$Y = 50 + (50/G ** 3) * (G - F) ** 3$$

For the special cases

G = 0 F ≠ 0 then Y = 0

G = 0 F = 0 then Y = 50

and when $0 < G \leq .5$ and $2G < F \leq 1$

$Y = 0$

Next consider the range $.5 < G < 1$

Whenever $F = G$, we want $Y = 50$ and when

$F = 2G - 1$, we want $Y = 100$.

Then imposing these two restrictions we have

$B(0) = 50$

$B(0) + B(3) * (G - 1) ** 3 = 100$

$B(3) = 50/(G - 1) ** 3$

Then we write the model for $.5 < G < 1$ and $(2G - 1) \leq F \leq 1$

$Y = 50 + (50/(1 - G) ** 3) * (G - F) ** 3$

For the special cases

$G = 1$ $F \neq 1$ then $Y = 100$

$G = 1$ $F = 1$ then $Y = 50$

and when $0 \leq F < (2G - 1)$

$Y = 100$

It is sometimes convenient to create a default option which maintains an on-target value of Y = 50 for all values of F. This is accomplished by setting Y = 50 when G = 0 or G = 1. If this procedure is used, it is necessary to assign G equal a small positive value near zero to represent a desired goal of zero, or to assign G equal a value slightly less than 1 to represent a desired goal of one.

APPENDIX D: MODEL DEVELOPMENT FOR EXAMPLE 4

Value to Air Force as a Function of Several Components

When several components are to be combined into a single composite, a policy maker may wish to control the "relative amount" that each component contributes to the composite. One approach to this is to convert the range of each variable into a fractional part of the composite.

Start with a model of N components of the form

$$Y = B(0) + B(1) * (X(1) - K(1)) + \ldots + B(I) * (X(I) - K(I)) + \ldots + B(N) * (X(N) - K(N))$$

There are N + 1 parameters to be determined. Since we wish to make statements of policy at high (H(I)) or low (L(I)) values of each variable X(I), we can arbitrarily set K(I) = L(I).

Then the starting model becomes

$$Y = B(0) + B(1) * (X(1) - L(1)) + \ldots + B(I) * (X(I) - L(I)) + \ldots + B(N) * (X(N) - L(N))$$

We would like the following conditions to be satisfied:

Let

HC = high value of composite

LC = low value of composite

F(I) = fraction of composite to be used by X(I)

Then we want

$$Y = LC \text{ when } X(I) = L(I) \qquad I = 1, \ldots, N$$

Substituting in the model to express this restriction we obtain

$$B(0) = LC$$

And when X(I) = H(I) and X(J) = L(J) for $J \neq I$

$$Y \text{ equals } LC + F(I)(HC - LC)$$

This implies

$$LC + B(I) * (H(I) - L(I)) = LC + F(I) * (HC - LC) \qquad I = 1, \ldots, N$$

Then we have

$$B(I) = (F(I) * (HC - LC))/(H(I) - L(I)) \qquad I = 1, \ldots, N$$

Substituting these values for B(0) and B(I) gives

$$Y = LC + ((F(1) * (HC - LC))/(H(1) - L(1))) * (X(1) - L(1)) + \ldots + ((F(I) * (HC - LC))/(H(I) - L(I))) * (X(I) - L(I)) + \ldots + (F(N) * (HC - LC))/(H(N) - L(N))) * (X(N) - L(N))$$

APPENDIX E: MODEL DEVELOPMENT FOR GENERALIZED MODEL 1

The major restrictions for Model 1 are that there be one and only one value of variable A (call it A(KONA)) for which the A-slope equals 0 at every value of D; and that there be one and only one value of variable D (call it D(KOND)) for which the D-slope equals 0 at every value of A.

The polynomial form of degrees AEXP and DEXP in A and D that has the above properties is

Y = B(1) + B(2) * (A - A(KONA)) * * AEXP
+ B(3) * (D - D(KOND)) * * DEXP
+ B(4) * ((A - A(KONA)) * * AEXP) * ((D - D(KOND)) * * DEXP)

The four unknown values B(1), . . . , B(4) can be determined by policy-specifying the Y values at four critical combinations of A and D.

The A values for control are designated

A(KONA) and A(KONACH)

and the D control values are

D(KOND) and D(KONDCH)

The policy statements give explicit values of Y for all four combinations of A and D control values. These four Y values are named

Y(KONA, KOND)
Y(KONA, KONDCH)
Y(KONACH, KOND)
Y(KONACH, KONDCH)

Then the four restrictions imposed on the model are

(R-1) B(1) = Y (KONA, KOND)

(R-2) B(1) + B(3) * (D(KONDCH) - D(KOND)) ** DEXP = Y(KONA, KONDCH)

(R-3) B(1) + B(2) * (A(KONACH) - A(KONA)) * * AEXP = Y (KONACH, KOND)

(R-4) B(1) + B(2) * (A(KONACH) - A(KONA)) * * AEXP
+ B(3) * (D(KONDCH) - D(KOND)) * * DEXP
+ B(4) * ((A(KONACH) - A(KONA)) * * AEXP) *
((D(KONDCH) - D(KOND)) * * DEXP) = Y (KONACH, KONDCH)

Solving these four restrictions we obtain from R-1

B(1) = Y(KONA, KOND)

from R-2

B(3) = (Y(KONA, KONDCH) - Y(KONA, KOND))/((D(KONDCH) - D(KOND)) * * DEXP)

from R-3

B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA)) * * AEXP)

and from R-4

B(4) = (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, KONDCH))/(((A(KONACH) - A(KONA)) * * AEXP) * ((D(KONDCH) - D(KOND)) * * DEXP))

APPENDIX F: MODEL DEVELOPMENT FOR GENERALIZED MODEL 2

The major restrictions for Model 2 are that there be one and only one value of variable A (call it A(KONA)) for which the A-slope equals 0 at every value of D (the same as Model 1); but there should be two and only two values of variable D (one of which is D(KOND)) for which the D-slope equals 0 at every value of A.

The polynomial form of degrees AEXP and DEXP in A and D that has the above properties is

Y = B(1) + B(2) * (A - A(KONA)) ** AEXP
+ B(3) * (D - D(KOND)) ** (DEXP - 1)
+ B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)
+ B(5) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** (DEXP - 1))
+ B(6) * (D - D(KOND)) ** DEXP

In order to control the movement of a maximum (or minimum) ridge (or valley) we will require two special restrictions on the D-slopes.

Statements 1 and 2; Restrictions 1 and 2

The starting model above already has D-slope equal zero at D(KOND). To control the ridge (or valley) movements we will require that

(1) When A = A(KONA), the D-slope = 0 at D = D(KONDCH) and that

(2) When A = A(KONACH), the D-slope = 0 only at D = D(KOND). One sketch of relationships that have these conditions is presented in Figure F-1.

Then we write the expression for first partial derivative of Y with respect to D.

Y1D = B(3) * (DEXP - 1) * (D - D(KOND)) ** (DEXP - 2)
+ B(4) * DEXP * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 1)
+ B(5) * (DEXP - 1) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 2)
+ B(6) * DEXP * (D - D(KOND)) ** (DEXP - 1)

then the two restrictions become,

when A = A(KONA) and D = D(KONDCH), Y1D = 0 or

(R-1) B(3) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 2) + B(6) * DEXP * (D(KONDCH) -
- D(KOND)) ** (DEXP - 1) = 0

and when A = A(KONACH) the D-slope = 0 only at D = D(KOND)

Rewriting and factoring (D - D(KOND)) ** (DEXP - 2)

Y1D = ((D - D(KOND)) ** (DEXP - 2)) * [B(3) * (DEXP - 1) + B(4) * DEXP * ((A(KONACH) - A(KONA) ** AEXP) * (D - D(KOND)) + B(5) * (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP
+ B(6) * DEXP * (D - D(KOND)]

Then we observe that the above expression is guaranteed to be zero only at D = D(KOND) by requiring that

(R-2) D(3) * (DEXP - 1) + B(5) * (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP = 0

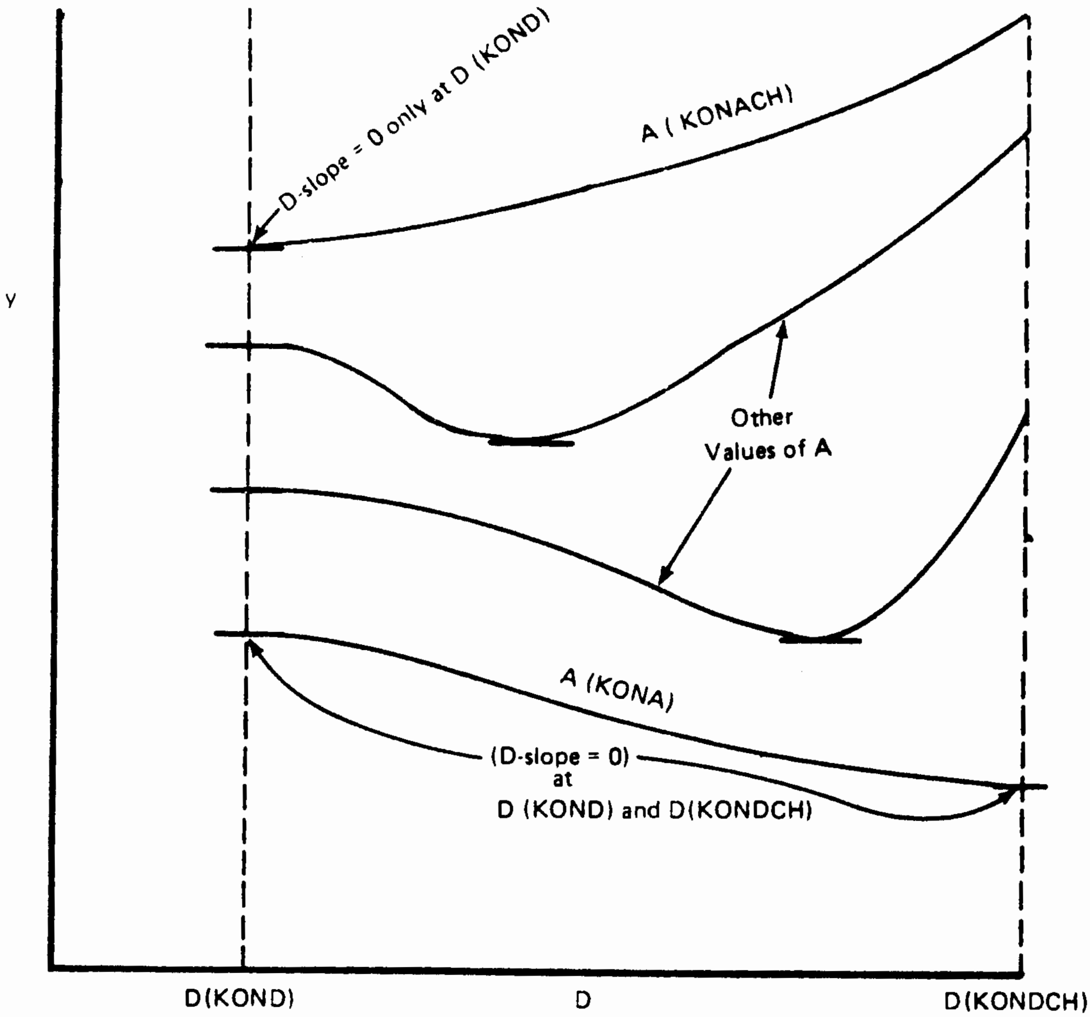

Figure F-1. **Sketch of conditions for D-slopes in generalized Model 2.**

Statements 3-6; Restrictions 3-6

The remaining four restrictions are provided by specifying (as in Model 1)

Y(KONA, KOND)

Y(KONA, KONDCH)

Y(KONACH, KOND)

Y(KONACH, KONDCH)

The four restrictions are

(R-3) B(1) = Y (KONA, KOND)

(R-4) B(1) + B(3) * (D(KONDCH) - D(KOND)) ** (DEXP - 1) + B(6) * (D(KONDCH) - D(KOND)) ** DEXP = Y(KONA, KONDCH)

(R-5) B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP = Y(KONACH, KOND)

(R-6) B(1) + B(2) * (A(KONACH) - A(KONA)) ** AEXP + B(3) * (D(KONDCH) - D(KOND)) ** (DEXP -1)
+ B(4) * ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)
+ B(5) * ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** (DEXP - 1))
+ B(6) * (D(KONDCH) - D(KOND) ** DEXP = Y(KONACH, KONDCH)

Now to simplify the development let

C(I,J) = coefficient of B(J) in the Ith restriction

then

C(1,3) = (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 2)

C(1,6) = (DEXP * (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(2,3) = (DEXP - 1)

C(2,5) = (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP

C(3,1) = C(4,1) = C(5,1) = C(6,1) = 1

C(4,3) = (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(4,6) = (D(KONDCH) - D(KOND)) ** DEXP

C(5,2) = (A(KONACH) - A(KONA)) ** AEXP

C(6,2) = (A(KONACH) - A(KONA)) ** AEXP

C(6,3) = (D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(6,4) = ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP

C(6,5) = ((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** (DEXP - 1)

C(6,6) = (D(KONDCH) - D(KOND)) ** DEXP

Now the restrictions can be summarized as follows:

	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)	
R-1			C(1,3)			C(1,6)	= 0
R-2			C(2,3)		C(2,5)		= 0
R-3	1						= Y(KONA, KOND)
R-4	1		C(4,3)			C(4,6)	= Y(KONA, KONDCH)
R-5	1	C(5,2)					= Y(KONACH, KOND)
R-6	1	C(6,2)	C(6,3)	C(6,4)	C(6,5)	C(6,6)	= Y(KONACH, KONDCH)

From R-3 we can obtain

B(1) = Y(KONA, KOND)

from R-5

B(2) = Y(KONCH, KOND) - Y(KONA, KOND))/C(5,2)

B(2) = (Y(KONCH, KOND) - Y (KONA, KOND))/((A(KONACH) - A(KONA)) ** AEXP)

from R-1 and R-4

B(3) * C(1,3) + B(6) * C(1,6) = 0

B(1) + B(3) * C(4,3) + B(6) * C(4,6) = Y(KONA, KONDCH)

Solving first for B(3) gives

B(3) = (C(1,6) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((C(4,3) * C(1,6)) - (C(1,3) * C(4,6)))

and after substitution

B(3) = (DEXP * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1))

Using R-2

B(3) * C(2,3) + B(5) * C(2,5) = 0

and

B(5) = (-B(3) * C(2,3))/C(2,5)

B(5) = (-B(3))/(A(KONACH) - A(KONA)) ** AEXP

Using R-1

B(3) * C(1,3) + B(6) * C(1,6) = 0

and

B(6) = (-B(3) * C(1,3))/C(1,6)

B(6) = ((-B(3)) * (DEXP - 1))/(DEXP * (D(KONDCH) - D(KOND)))

Using R-6

B(1) + B(2) * C(6,2) + B(3) * C(6,3) + B(4) * C(6,4) + B(5) * C(6,5) + B(6) * C(6,6) = Y(KONACH, KONDCH)

Solving for B(4) gives

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + ((DEXP - 1) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP))

APPENDIX G: MODEL DEVELOPMENT FOR GENERALIZED MODEL 3

The only difference between Model 2 and this model is that restrictions 1 and 2 in this model will control inflection points rather than D-slope = 0.

The polynomial form to start is as in Model 2

Y = B(1) + B(2) * (A - A(KONA)) ** AEXP
+ B(3) * (D - D(KOND)) ** (DEXP - 1)
+ B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)
+ B(5) * ((A - A(KONA)) ** AEXP) * (DD - D(KOND)) ** (DEXP - 1))
+ B(6) * (D - D(KOND)) ** DEXP

Statements 1 and 2; Restrictions 1 and 2

To control the inflection points we will write the expression for second partial derivative of Y with respect to D.

Y2D = B(3) * (DEXP - 2) * (DEXP - 1) * (D - D(KOND)) ** (DEXP - 3)
+ B(4) * (DEXP - 1) * (DEXP) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 2)
+ B(5) * (DEXP - 2) * (DEXP - 1) * ((A - A(KONA)) ** AEXP) * (D - D(KOND)) ** (DEXP - 3)
+ B(6) * (DEXP - 1) * (DEXP) * (D - D(KOND)) ** (DEXP - 2)

Then the restriction for inflection points control are

When A = A(KONA) and D = D(KONDCH), Y2D = 0 or

(R-1) B(3) * (DEXP - 2) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 3) + B(6) * (DEXP - 1) * (DEXP) * (D(KONDCH) - D(KOND)) ** (DEXP - 2) = 0

and when A = A(KONACH), Y2D = 0 only at D = D(KOND)

Rewriting and factoring (D - D(KOND)) ** (DEXP - 3)

Y2D = ((D - D(KOND)) ** (DEXP - 3)) * (B(3) * (DEXP - 2) * (DEXP - 1)
+ B(4) * (DEXP - 1) * DEXP * ((A(KONACH) - A(KONA)) ** AEXP) * (D - D(KOND))
+ B(5) * (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA) ** AEXP
+ B(6) * (DEXP - 1) * (DEXP * (D - D(KOND))

Then we observe that the above expression is guaranteed to be zero only at D = D(KOND) by requiring that

(R-2) B(3) * (DEXP - 2) * (DEXP - 1) + B(5) * (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA)) ** AEXP = 0

Then the remaining four restrictions are the same as in Model 2.

Now we can let

D(1,3) = (DEXP - 2) * (DEXP - 1) * (D(KONDCH) - D(KOND)) ** (DEXP - 3)

But noticing the value of C(1,3) and C(1,6) in Model 2 (Appendix F)

D(1,3) = C(1,3) * ((DEXP - 2)/(D(KONDCH) - D(KOND)))

and

D(1,6) = (DEXP - 1) * (DEXP) * (D(KONDCH) - D(KOND)) ** (DEXP - 2)

D(1,6) = C(1,6) * ((DEXP - 1)/(D(KONDCH) - D(KOND))) and

D(2,3) = (DEXP - 2) * (DEXP - 1)

D(2,3) = C(2,3) * (DEXP - 2)

Furthermore,

D(2,5) = (DEXP - 2) * (DEXP - 1) * (A(KONACH) - A(KONA) ** AEXP

D(2,5) = C(2,5) * (DEXP - 2)

Then the solutions for B(1), B(2), and B(5) are the same as for Model 2 and we have new values for B(3), B(4), and B(6)

B(3) = (D(1,6) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((C(4,3) * D(1,6)) - (D(1,3) * C(4,6)))

and after substitution

B(3) = ((DEXP/2) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1)

and

B(6) = ((-B(3)) * D(1,3))/D(1,6)

B(6) = ((-B3)) * (DEXP - 2))/(DEXP * (D(KONDCH) - D(KOND)))

Using restriction (R-6) gives

B(4) = (Y(KONACH), KONDCH) - Y(KONACH, KOND) + (((DEXP - 2)/2) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)

APPENDIX H: COMBINING MODEL 2 AND MODEL 3

By observing the close similarity between Model 2 and Model 3 we can insert a new parameter

MOD = 2 if Model 2
or 3 if Model 3

Then the coefficients for Model 2 or Model 3 can be determined by setting MOD = 2 or 3 in B(3), B(4), and B(6). B(1), B(2), and B(5) are the same in both models.

B(1) = Y(KONA, KOND)

B(2) = (Y(KONACH, KOND) - Y(KONA, KOND))/((A(KONACH) - A(KONA) ** AEXP

B(3) = (DEXP/(MOD - 1)) * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) ** (DEXP - 1)

B(4) = (Y(KONACH, KONDCH) - Y(KONACH, KOND) + (((DEXP - MOD + 1/(MOD - 1)) * (Y(KONA, KONDCH) - Y(KONA, KOND))))/(((A(KONACH) - A(KONA) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)

B(5) = (-B(3))/(A(KONACH) - A(KONA)) ** AEXP

B(6) = ((-B(3)) * (DEXP - MOD + 1))/(DEXP * (D(KONDCH) - D(KOND)))

APPENDIX I: MODELS 1, 2, AND 3 FORTRAN PROGRAM ON UNIVAC 1108

The following program provides interactive control of the parameters for Models 1, 2, and 3. At the end of an interactive session the user can request a copy of the output on the high-speed printer.

The program is entered on the UNIVAC system by the command @XQT CS*06PROG.MODELS–4X/25MAY. A sample execution of the program follows the program listing.

```
      DIMENSION YY(40,40),IFMT(20),KYY(40,40)
      INTEGER Y(2,2)/15,35,-250,100/A(2)/40,95/D(2)/40,100/AEXP/1/
     1DEXP/3/KONA/2/KOND/1/KSTARA/100/KSTOPA/35/KINCA/-5/
     2KSTARD/35/KSTOPD/105/KINCD/5/YES/1HY/BLANK/4H    /NO/1HN/
     3DIF(3)/4H   D,4H   I,4H   F/APT(3)/1HA,1HP,1HT/
15    FORMAT(A1)
3     FORMAT(' Y = ',I4,' + [<',E9.4,'>*(A - ',I3,')**',I2,'] + [<',
     1E9.4,'>*(D - ',I3,')**',I2,']',/,T11,'+ [<',E9.4,'>*[(A - ',I3,
     2')**',I2,
     3']*[(D - ',I3,')**',I2,']]',//)
2     FORMAT(' Y = ',I4,' + [<',E9.4,'>*(A - ',I3,')**',I2,'] + [<',
     1E9.4,'>*(D - ',I3,')**',I2,']',/,T11,'+ [<',E9.4,'>*[(A - ',I3,
     2')**'
     3,I2,']*[(D - ',I3,')**',I2,']]',/,T11,'+ [<',E9.4,'>*[(A - ',I3,
     4')**',I2,']*[(D - ',I3,')**',I2,']]',/,T11,'+ [<',E9.4,'>*(D - '
     5,I3,')**',I2,']',//)
30    FORMAT(' DO YOU WISH TO CHANGE PARAMETERS? (YES OR NO)')
45    FORMAT(' DO YOU WISH TO CHANGE Y? (YES OR NO)')
60    FORMAT(' Y(1,1)=?,Y(2,1)=?,Y(1,2)=?,Y(2,2)=?,')
75    FORMAT(' DO YOU WISH TO CHANGE A OR D? (YES OR NO)')
90    FORMAT(' A(1)=?,A(2)=?,D(1)=?,D(2)=?,')
99    FORMAT ()
100   FORMAT(' IS THIS YOUR FINAL CHANGE? (YES OR NO) ')
105   FORMAT(' DO YOU WISH TO CHANGE EXP OR CONT? (YES OR NO)')
120   FORMAT (' AEXP=?,DEXP=?,KONA=?,KOND=?,')
130   FORMAT (' DO YOU WISH TO CHANGE OUTPUT? (YES OR NO)')
160   FORMAT(' PARAMETER ERROR  A(1)=A(2) OR D(1)=D(2)')
145   FORMAT(' KSTARA=?,KSTOPA=?,KINCA=?,KSTARD=?,KSTOPD=?,KINCD=?,')
175   FORMAT('1MODEL=',I1,//,
     1' Y(1,1)=',I4,' Y(2,1)=',I4,' Y(1,2)=',I4,' Y(2,2)=',I4/
     2' AEXP=',I2,' DEXP=',I2,' KONA=',I2,' KOND=',I2,/,
     3' A(1)=',I3,' A(2)=',I3,' D(1)=',I3,' D(2)=',I3/,
     4' KSTARA=',I3,
     5' KSTOPA=',I3,' KINCA=',I3/' KSTARD=',I3,' KSTOPD=',I3,
     5' KINCD=',I3//)
190   FORMAT(T7,20A4)
205   FORMAT(/,T7,20I4)
220   FORMAT(1H ,A1,20I4)
235   FORMAT(///' DO YOU WISH TO WORK ANOTHER PROBLEM? (YES OR NO)'///)
```

```
236   FORMAT(///' DO YOU WISH HARDCOPY? (YES OR NO)'///)
250   FORMAT(' PLEASE SELECT A MODEL (1 OR 2 OR 3)')
265   FORMAT(' DEXP-MODEL+1 MUST BE NON-NEGATIVE'/' MODEL=',
     1I1,' DEXP=',I2,' PLEASE CHANGE PARAMETER DEXP'/)
266   FORMAT(' DEXP=?')
C**********THE FOLLOWING TWO STATEMENTS TEST TO SEE IF FILE 6 IS
C**********    ALREADY CATALOGUED -- TO AVOID BLOW UP OF PROGRAM
      I=NERTRN (6,'@ASG,C 6. . ')
      IF (I. LT . 0)  CALL ERTRAN (6,'@ASG,A 6. . ')
1     PRINT 250
      READ(5,99) MODEL
      IF(MODEL.EQ.1)GO TO 18
      IF(DEXP-MODEL+1.LT.0)PRINT 265,MODEL,DEXP
18    PRINT 30
      READ(5,15) KPARAM
      IF (KPARAM .EQ. NO) GO TO 115
      IF (KPARAM .NE. YES) GO TO 18
19     PRINT 45
      READ(5,15) KPARAM
      IF (KPARAM .EQ. NO) GO TO 215
      IF (KPARAM .NE. YES) GO TO 19
      PRINT 60
      READ(5,99) Y
20    PRINT 100
      READ(5,15) KPARAM
      IF(KPARAM.EQ.YES) GO TO 115
      IF(KPARAM .NE. NO) GO TO 20
215   PRINT 105
      READ(5,15) KPARAM
      IF(KPARAM .EQ. NO) GO TO 315
      IF(KPARAM.NE.YES)GO TO 215
      PRINT 120
      READ(5,99) AEXP,DEXP,KONA,KOND
21    PRINT 100
      READ(5,15) KPARAM
      IF(KPARAM.EQ.YES) GO TO 115
```

```
      IF(KPARAM .NE. NO) GO TO 21
315   PRINT 75
      READ(5,15) KPARAM
      IF(KPARAM .EQ. NO) GO TO 415
      IF (KPARAM .NE. YES) GO TO 315
      PRINT 90
      READ(5,99) A,D
22    PRINT 100
      READ(5,15) KPARAM
      IF(KPARAM.EQ.YES) GO TO 115
      IF(KPARAM .NE. NO) GO TO 22
415   PRINT 130
      READ(5,15) KPARAM
      IF(KPARAM .EQ. NO) GO TO 115
      IF (KPARAM .NE. YES) GO TO 415
      PRINT 145
      READ(5,99) KSTARA,KSTOPA,KINCA,KSTARD,KSTOPD,KINCD
115   CONTINUE
      IF (MODEL.EQ.1) GO TO 114
119   IF (DEXP-MODEL+1.GE.0) GO TO 114
      PRINT 265, MODEL,DEXP
      PRINT 266
      READ(5,99) DEXP
      GO TO 119
114   IFMT(1)='(/,T7,'
      LF=1
      DO 777 LC=KSTARD,KSTOPD,KINCD
      LF=LF+1
777   IFMT(LF)='I4,'
      IFMT(LF+1)='/)'
      IF(A(3-KONA).EQ.A(KONA).OR.D(3-KOND).EQ.D(KOND)) GO TO 515
      Y0=Y(KONA,KOND)
      Y1=Y(3-KONA,KOND)
      Y2=Y(KONA,3-KOND)
      Y3=Y(3-KONA,3-KOND)
      A0=(A(3-KONA)-A(KONA))**AEXP
      D0=D(3-KOND)-D(KOND)
      W1=(Y1-Y0)/A0
```

```
      IF(MODEL-2) 101,102,102
101   W2=(Y2-Y0)/(D0**DEXP)
      W3=(Y0-Y2-Y1+Y3)/(A0*(D0**DEXP))
      GO TO 103
102   W2=(DEXP*(Y2-Y0))/((MODEL-1)*(D0**(DEXP-1)))
      W3=(Y3-Y1+(FLOAT(DEXP-MODEL+1)/(MODEL-1))*(Y2-Y0))/((D0**DEXP)*A0)
      W4=-W2/A0
      W5=-W2*(DEXP-MODEL+1)/(DEXP*D0)
103   CONTINUE
      LF=0
      DO 500 LC=KSTARA,KSTOPA,KINCA
      LF=LF+1
      LS=0
      DO 500 LL=KSTARD,KSTOPD,KINCD
      LS=LS+1
      U0=(LC-A(KONA))**AEXP
      U1=LL-D(KOND)
      S1=Y0+W1*U0
      IF(MODEL.LT.2) S2=S1+W2*U1**DEXP
      IF(MODEL.GE.2)
     1S2=S1+W2*(U1**(DEXP-1))*(1-(U0/A0)-(((DEXP-MODEL+1)*U1)/
     1(DEXP*D0)))
      YY(LF,LS)=S2+W3*(U1**DEXP)*U0
500   CONTINUE
      GO TO 615
515   PRINT 160
      GO TO 616
615   WRITE(6,175) MODEL,Y,AEXP,DEXP,KONA,KOND,A,D,KSTARA,
     1KSTOPA,KINCA,KSTARD,KSTOPD,KINCD
      PRINT 175,MODEL,Y,AEXP,DEXP,KONA,KOND,A,D,KSTARA,
     1KSTOPA,KINCA,KSTARD,KSTOPD,KINCD
      IDEXP=DEXP-1
      IF(MODEL .GT. 1)  GO TO 625
            WRITE(6,3)Y(KONA,KOND),W1,A(KONA),AEXP,W2,D(KOND),
     1DEXP,W3,A(KONA),AEXP,D(KOND),DEXP
         PRINT 3,  Y(KONA,KOND),W1,A(KONA),AEXP,W2,D(KOND),
     1DEXP,W3,A(KONA),AEXP,D(KOND),DEXP
      GO TO 630
```

```
625       WRITE(6,2)Y(KONA,KOND),W1,A(KONA),AEXP,W2,D(KOND),
         1IDEXP,W3,A(KONA),AEXP,D(KOND),DEXP,W4,A(KONA),AEXP,D(KOND),
         2IDEXP,W5,D(KOND),DEXP
          PRINT 2,  Y(KONA,KOND),W1,A(KONA),AEXP,W2,D(KOND),
         1IDEXP,W3,A(KONA),AEXP,D(KOND),DEXP,W4,A(KONA),AEXP,D(KOND),
         2IDEXP,W5,D(KOND),DEXP
630       CONTINUE
          KKD=((KSTOPD-KSTARD)/KINCD+1)/2
          KKA=((KSTOPA-KSTARA)/KINCA+1)/2
          KKDM1 = KKD-1
          WRITE(6,190)(BLANK,LM=1,KKDM1),DIF
          PRINT 190,(BLANK,LM=1,KKDM1),DIF
          WRITE(6,IFMT)(LC,LC=KSTARD,KSTOPD,KINCD)
          PRINT IFMT,(LC,LC=KSTARD,KSTOPD,KINCD)
          LF=0
          LL=(KSTOPD-KSTARD)/KINCD+1
          DO 600 LC=KSTARA,KSTOPA,KINCA
          LF=LF+1
          IB=BLANK
          IF(LF.GE.KKA.AND.LF.LE.KKA+2) IB=APT(LF-KKA+1)
          DO 3141 LS=1,LL
          IF (YY(LF,LS) .LT. 0 )   YY(LF,LS)=YY(LF,LS)-1.0
3141      KYY(LF,LS) = YY(LF,LS) + .5
          WRITE(6,220) IB,LC,(KYY(LF,LS),LS=1,LL)
          PRINT 220, IB,LC,(KYY(LF,LS),LS=1,LL)
600       CONTINUE
616       CONTINUE
          PRINT 235
          READ(5,15) KPARAM
          IF(KPARAM.EQ.YES) GO TO 1
          IF(KPARAM .NE. NO) GO TO 616
23        PRINT 236
          READ(5,15) KPARAM
          IF (KPARAM .EQ. NO) GO TO 237
          IF (KPARAM .NE. YES) GO TO 23
          CALL ERTRAN(6,'@BRKPT 6 . ')
          CALL ERTRAN(6,'@FREE 6. . ')
          CALL ERTRAN (6,'@SYM 6. . ')
          STOP
237       CALL ERTRAN(6,'@FREE,I 6. . ')
          STOP
          END
```

```
USAF01
*UNIVAC 1100 OPERATING SYSTEM  VER. 32-R2B-18C  (RSI)*
@RUN CS12,20770401,CS
DATE: 041277        TIME: 082017
>@XQT 06PROG.MODELS-4X/25MAY
 PLEASE SELECT A MODEL (1 OR 2 OR 3)
>1
 DO YOU WISH TO CHANGE PARAMETERS? (YES OR NO)
>NO
```

```
 MODEL=1

 Y(1,1)=   15 Y(2,1)=   35 Y(1,2)=-250 Y(2,2)= 100
 AEXP= 1 DEXP= 3 KONA= 2 KOND= 1
 A(1)= 40 A(2)= 95 D(1)= 40 D(2)=100
 KSTARA=100 KSTOPA= 35 KINCA= -5
 KSTARD= 35 KSTOPD=105 KINCD=  5


 Y =    35 + [( .3636+00)*(A -   95)** 1] + [( .3009-03)*(D -   40)** 3]
           + [( .2778-04)*[(A -   95)** 1]*[(D -   40)** 3]]
```

D I F

		35	40	45	50	55	60	65	70	75	80	85	90	95	100	105
	100	37	37	37	37	38	40	44	49	56	65	77	92	110	132	158
	95	35	35	35	35	36	37	40	43	48	54	62	73	85	100	118
	90	33	33	33	33	34	34	36	38	40	44	48	53	60	68	78
	85	31	31	31	31	31	32	32	32	32	33	33	34	35	36	38
	80	30	30	30	29	29	29	28	26	25	22	19	15	10	5	-2
	75	28	28	28	27	27	26	24	21	17	11	5	-4	-15	-27	-42
A	70	26	26	26	26	25	23	20	15	9	1	-10	-23	-40	-59	-82
F	65	24	24	24	24	22	20	16	10	1	-10	-24	-42	-64	-91	-122
T	60	22	22	22	22	20	17	12	4	-7	-21	-39	-62	-89	-123	-162
	55	21	20	20	20	18	14	8	-1	-14	-31	-53	-81	-114	-155	-202
	50	19	19	19	18	15	11	4	-7	-22	-42	-68	-100	-139	-186	-242
	45	17	17	17	16	13	8	0	-13	-30	-53	-82	-119	-164	-218	-282
	40	15	15	15	14	11	5	-4	-18	-38	-64	-97	-138	-189	-250	-322
	35	13	13	13	12	9	2	-8	-24	-45	-74	-111	-158	-214	-282	-362

```
 DO YOU WISH TO WORK ANOTHER PROBLEM? (YES OR NO)
```

```
>Y
 PLEASE SELECT A MODEL (1 OR 2 OR 3)
>1
 DO YOU WISH TO CHANGE PARAMETERS? (YES OR NO)
>Y
 DO YOU WISH TO CHANGE Y? (YES OR NO)
>Y
 Y(1,1)=?,Y(2,1)=?,Y(1,2)=?,Y(2,2)=?,
>10,100,0,100,
 IS THIS YOUR FINAL CHANGE? (YES OR NO)
>N
 DO YOU WISH TO CHANGE EXP OR CONT? (YES OR NO)
>Y
 AEXP=?,DEXP=?,KONA=?,KOND=?,
>5,1,1,1,
 IS THIS YOUR FINAL CHANGE? (YES OR NO)
>N
 DO YOU WISH TO CHANGE A OR D? (YES OR NO)
>Y
 A(1)=?,A(2)=?,D(1)=?,D(2)=?,
>0,21,0,10,
 IS THIS YOUR FINAL CHANGE? (YES OR NO)
>N
 DO YOU WISH TO CHANGE OUTPUT? (YES OR NO)
>Y
 KSTARA=?,KSTOPA=?,KINCA=?,KSTARD=?,KSTOPD=?,KINCD=?,
>21,0,-1,0,10,1,
 MODEL=1
 Y(1,1)=   10 Y(2,1)= 100 Y(1,2)=   0 Y(2,2)= 100
 AEXP= 5 DEXP= 1 KONA= 1 KOND= 1
 A(1)=   0 A(2)= 21 D(1)=   0 D(2)= 10
 KSTARA= 21 KSTOPA=   0 KINCA= -1
 KSTARD=   0 KSTOPD= 10 KINCD=   1
 Y =    10 + [( .2204-04)*(A -     0)** 5] + [(-.1000+01)*(D -     0)** 1]
          + [( .2449-06)*[(A -     0)** 5]*[(D -     0)** 1]]
```

D I F

		0	1	2	3	4	5	6	7	8	9	10
	21	100	100	100	100	100	100	100	100	100	100	100
	20	81	80	80	80	80	79	79	79	79	79	78
	19	65	64	64	63	63	63	62	62	61	61	61
	18	52	51	51	50	49	49	48	48	47	47	46
	17	41	41	40	39	39	38	37	37	36	35	35
	16	33	32	32	31	30	29	29	28	27	26	26
	15	27	26	25	24	23	23	22	21	20	19	19
	14	22	21	20	19	18	18	17	16	15	14	13
	13	18	17	16	15	15	14	13	12	11	10	9
	12	15	15	14	13	12	11	10	9	8	7	6
A	11	14	13	12	11	10	9	8	7	6	5	4
P	10	12	11	10	9	8	7	6	5	4	3	2
T	9	11	10	9	8	7	6	5	4	3	2	1
	8	11	10	9	8	7	6	5	4	3	2	1
	7	10	9	8	7	6	5	4	3	2	1	0
	6	10	9	8	7	6	5	4	3	2	1	0
	5	10	9	8	7	6	5	4	3	2	1	0
	4	10	9	8	7	6	5	4	3	2	1	0
	3	10	9	8	7	6	5	4	3	2	1	0
	2	10	9	8	7	6	5	4	3	2	1	0
	1	10	9	8	7	6	5	4	3	2	1	0
	0	10	9	8	7	6	5	4	3	2	1	0

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