

Values of y are presented in Table 5 for various combinations of x_1 and x_2 .

Table 5. Y (Payoff) = Function of x_1 and x_2 ($y = 6x_1 + 8x_2$)

		x_1										
		0	10	20	30	40	50	60	70	80	90	100
x_2	50	400	460	520	580	640	700	760	820	880	940	1000
	40	320	380	440	500	560	620	680	740	800	860	920
	30	240	300	360	420	480	540	600	660	720	780	840
	20	160	220	280	340	400	460	520	580	640	700	760
	10	80	140	200	260	320	380	440	500	560	620	680
	0	0	60	120	180	240	300	360	420	480	540	600

IV. GENERAL PURPOSE MODELS FOR POLICY-SPECIFYING

It was indicated in Section I that policy-specifying required repeated creation of new models with different properties in an effort to produce output values that are acceptable to the policy maker. Section II (and the corresponding Appendixes A through D) gave specific examples of models with interactions among various powers of variables. If the processes, described in Appendixes A through D, had to be repeated everytime a new model was desired, each policy-specifying cycle would be extremely slow. This repetitious and slow process of imposing restrictions for each different situation provides good practice for the model maker but it slows and impedes (and could possibly destroy) the policy-specifying process. To make policy-specifying a viable approach for representing value judgments, several general forms were developed. By varying parameter settings it is easy to generate many different models and examine the results quickly. This section contains the three general forms that were developed with examples of how they can be used to create specific models. At the end, the three general forms are presented together to provide a pictorial aid to policy-specifying. In this section all policy-specified models are written using FORTRAN expressions. This provides for accurate communication of the models and ease of implementation on a computer.

Definitions

The parameters that will be used to describe the general models are described as follows. Values for these parameters are determined by policy. Referring to the following information.

- A(1) = the smaller control value of variable A
- A(2) = the larger control value of variable A
- D(1) = the smaller control value of variable D
- D(2) = the larger control value of variable D
- Y(1,1) = the Y value corresponding to A(1), D(1)
- Y(2,1) = the Y value corresponding to A(2), D(1)
- Y(1,2) = the Y value corresponding to A(1), D(2)
- Y(2,2) = the Y value corresponding to A(2), D(2)
- AEXP = the polynomial exponent for variable A

DEXP = the polynomial exponent for variable D
 KONA = control for variable A characteristics
 KOND = control for variable D characteristics. KONA and KOND are used to specify the locations and movements of slopes and inflection points. KONA will take the value of 1 to specify control with reference to A(1) and the value 2 to specify control with reference to A(2). Similarly KOND will take the value of 1 to specify control with reference to D(1) and the value 2 to specify control with reference to D(2).

An example of locations for the A's, D's, and Y's is presented as Figure 14.

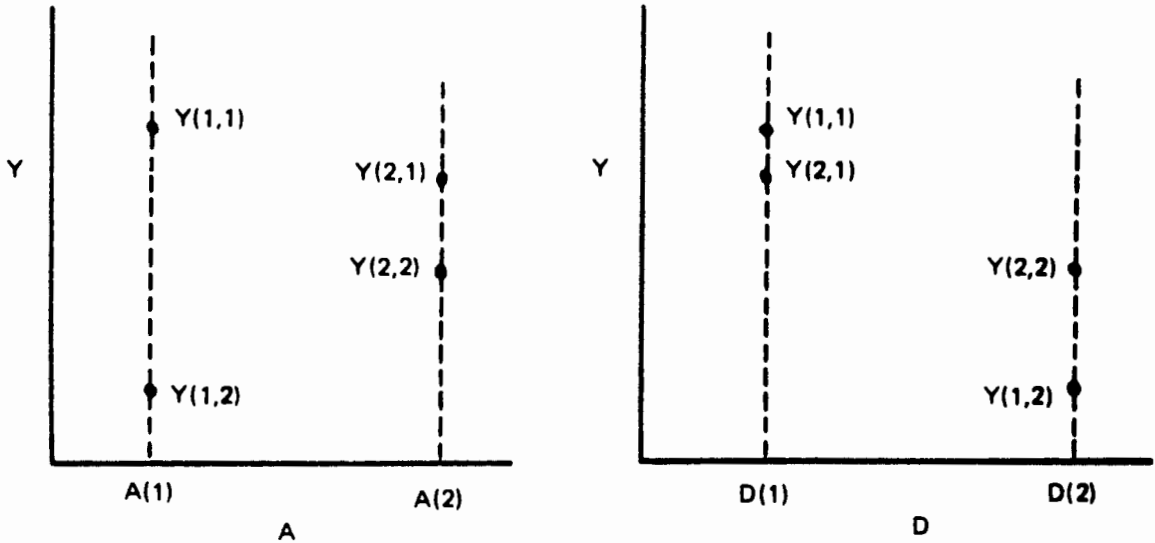


Figure 14. Y (payoff) = function of A and D. An example of locations of parameters.

Model 1

The first general model allows for expressing the composite value Y as a function of any two variables, A and D, using a general polynomial form with easy control of the location of slopes of zero and critical Y values. Appendix E describes the development of this model.

Model 1 is expressed as

$$Y = B(1) + B(2) * (A - A(KONA)) ** AEXP + B(3) * (D - D(KOND)) ** DEXP + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP)$$

Where

$$\begin{aligned} B(1) &= Y(KONA, KOND) \\ B(2) &= (Y(KONACH, KOND) - Y(KONA, KOND)) / ((A(KONACH) - A(KONA)) ** AEXP) \\ B(3) &= (Y(KONA, KONDCH) - Y(KONA, KOND)) / ((D(KONDCH) - D(KOND)) ** DEXP) \\ B(4) &= (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, KONDCH)) / (((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) ** DEXP)) \end{aligned}$$

Also note that * means "multiplication" and ** means "exponentiation."

Where KONACH = 3 - KONA
 = 2 when KONA = 1
 = 1 when KONA = 2
 KONDCH = 3 - KOND
 = 2 when KOND = 1
 = 1 when KOND = 2

This model has the following characteristics under control by KONA and KOND:

1. For every value of D the slope for variable A is zero at A(KONA) and no other values of A.
2. For every value of A the slope for variable D is zero at D(KOND) and no other values of D.
3. For any value of AEXP and for all values of D the variable A and the slope Y with respect to A (called A-slope) is either always increasing or always decreasing between A(1) and A(2). Note that higher values of AEXP result in slower changes in Y as A changes near A(KONA) and faster changes in Y as A changes near A(KONACH).
4. For any value of DEXP and for all values of A, the variable D and the slope of Y with respect to D (called D-slope) is either always increasing or always decreasing between D(1) and D(2). Note that higher values of DEXP result in slower changes in Y as D changes near D(KOND) and faster changes in Y as D changes near D(KONDCH).

Figure 15 shows sketches of curves at the critical control values for a possible policy specification.

This general form will now be used to represent Example 2 above where Y is a function of time used, T; fraction of fill, F; and job-importance, K.

In this situation the variable A corresponds to T (days used) and variable D corresponds to F (fraction of fill). As defined before, K is the job importance indicator. Then we can specify the parameters in the general model to obtain the specific model.

A(1)	=	0, the smaller control value of T (days used)
A(2)	=	180, the larger control value of T (days used)
D(1)	=	0, the smaller control value of F (fraction fill)
D(2)	=	1.0, the larger control value of F (fraction fill)
Y(1,1)	=	K, Y value at T = 0, F = 0
Y(2,1)	=	100, Y value at T = 180, F = 0
Y(1,2)	=	0, Y value at T = 0, F = 1.0
Y(2,2)	=	K, Y value at T = 180, F = 1.0
AEXP	=	1, the change in Y is constant as T changes
DEXP	=	1, the change in Y is constant as F changes
KONA	= 1	} Setting of KONA and KOND can be either 1 or 2, since there is no requirement of control for slopes.
KOND	= 1	
KONACH	=	3 - KONA = 2
KONDCH	=	3 - KOND = 2

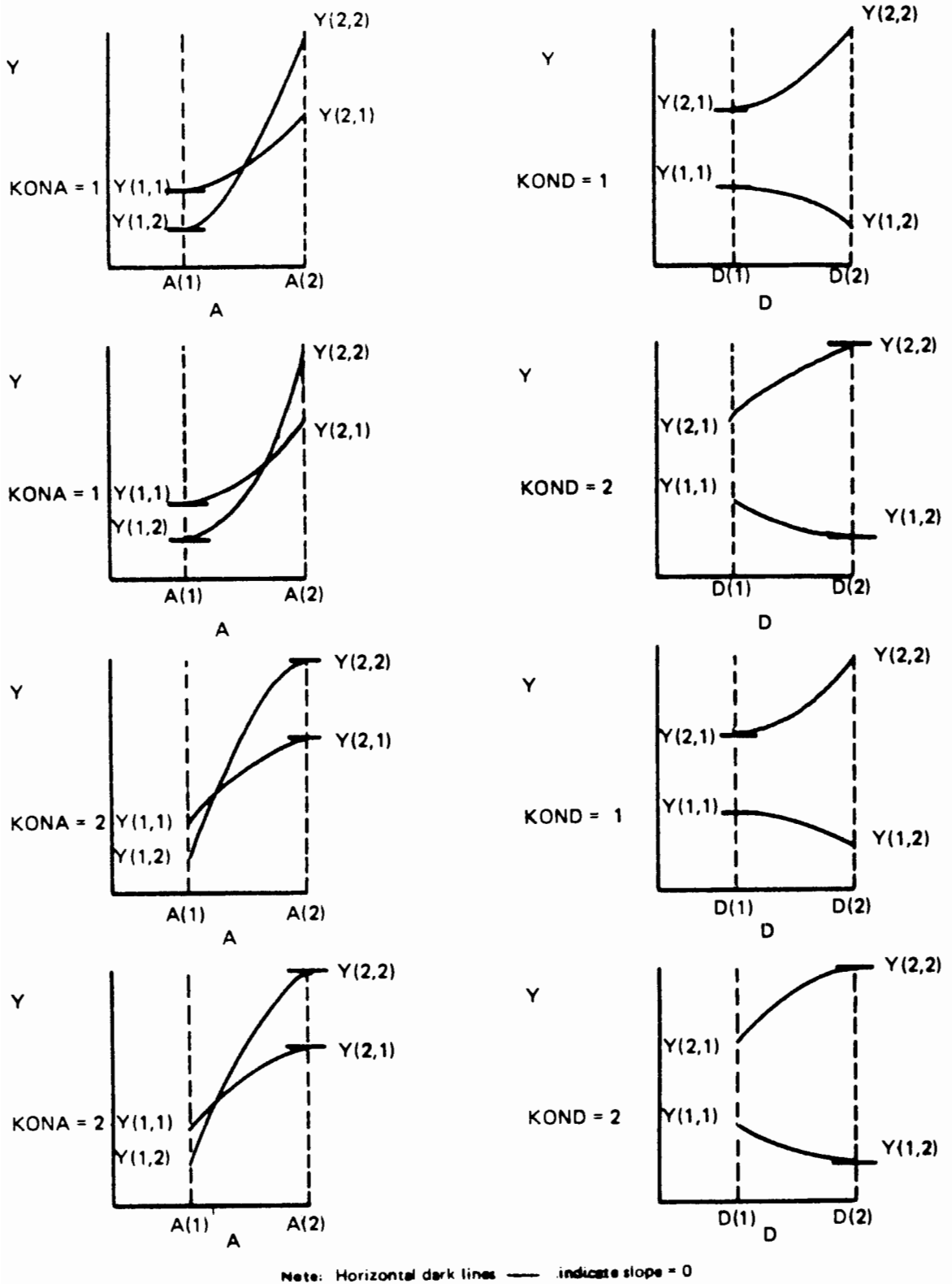


Figure 15. Y (payoff) = function of A and D . Examples of parameter settings for Model 1.

Substitution yields

$$\begin{aligned}
 B(1) &= Y(KONA, KOND) = Y(1,1) = K \\
 B(2) &= (Y(KONACH, KOND) - Y(KONA, KOND)) / ((A(KONACH) - A(KONA)) ** AEXP) \\
 &= (Y(2,1) - Y(1,1)) / ((A(2) - A(1)) ** 1) \\
 &= (100 - K) / (180 - 0) ** 1 = (100 - K) / 180 \\
 B(3) &= (Y(KONA, KONDCH) - Y(KONA, KOND)) / ((D(KONDCH) - D(KOND)) ** DEXP) \\
 &= (Y(1,2) - Y(1,1)) / ((D(2) - D(1)) ** 1) \\
 &= (0 - K) / (1.0 - 0) ** 1 = -K \\
 B(4) &= (Y(KONA, KOND) - Y(KONA, KONDCH) - Y(KONACH, KOND) + Y(KONACH, \\
 &KONDCH)) / (((A(KONACH) - A(KONA)) ** AEXP) * ((D(KONDCH) - D(KOND)) \\
 &** DEXP)) \\
 &= (Y(1,1) - Y(1,2) - Y(2,1) + Y(2,2)) / (((A(2) - A(1)) ** 1) * ((D(2) - D(1)) ** 1)) \\
 &= (K - 0 - 100 + K) / (((180 - 0) ** 1) * ((1.0 - 0) ** 1)) \\
 &= (2 * K - 100)
 \end{aligned}$$

and the final model written similar to the original form of Example 2 on page 16 is

$$Y = K + \frac{(100 - K)T}{180} + (-K)F + \frac{(2K - 100)(TF)}{180}$$

Recalling that $Y(1,1) = K$ and $Y(2,2) = K$, to generate this function when $K = 25$, requires $Y(1,1) = 25$ and $Y(2,2) = 25$. The output of this general model and its parameter settings are shown in Table 6. The results are identical to Table 2. The following parameters are used to control the range of A and D used in generating Y values from the function.

- KSTARA = the first value of A
- KSTOPA = the last value of A
- KINCA = the amount that A is incremented
- KSTARD = the first value of D
- KSTOPD = the last value of D
- KINCD = the amount that D is incremented

The parameter settings and output of this function for $K = 75$ is shown as Table 7. This output is the same as Table 3.

Now assume that the policy maker decides that everything is fine except that instead of constant changes in Y values the change in Y values should be gradual near $T = 0$ and $F = 0$ (corresponding to $A(1)$ and $D(1)$). This requires control of slopes and is accomplished by introducing a polynomial of degree 2 or more and by requiring the slopes of Y with respect to A (A-slope) for all D to be zero at $A(1)$ and the slope of Y with respect to D (D-slope) for all A to be zero at $D(1)$.

- Assume then that
- AEXP = 2
- DEXP = 2
- KONA = 1 (makes slopes = 0 at $A(1) = 0$)
- KOND = 1 (makes slopes = 0 at $D(1) = 0$)

The output of this new function for $K = 25$ and $K = 75$ is shown as Tables 8 and 9.

Table 6. Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 25

MODEL=1

Y(1,1)= 25 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 25
 AEXP= 1 DEXP= 1 KONA= 1 KOND= 1
 A(1)= 0 A(2)=180 J(1)= 0 D(2)=100
 KSTARA=180 KSTOPA= 0 KINCA=-15
 KSTARD= 0 KSTOPD=100 KINCD= 10

Y = 25 + [< .4167+00 > * (T - 0) ** 1] + [< -25.00 +00 > * (F - 0) ** 1]
 + [< -0.2778-00 > * (T - 0) ** 1] * [(F - 0) ** 1]

F I L L

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	93	85	78	70	63	55	48	40	33	25
165	94	87	80	73	65	58	51	44	37	30	23
150	87	81	74	68	61	54	48	41	34	28	21
135	81	75	69	63	56	50	44	37	31	25	19
120	75	69	63	58	52	46	40	34	28	22	17
105	69	63	58	52	47	42	36	31	25	20	15
90	62	58	53	48	43	38	33	28	23	18	13
75	56	52	47	43	38	33	29	24	20	15	10
60	50	46	42	38	33	29	25	21	17	12	8
45	44	40	36	33	29	25	21	17	14	10	6
30	37	34	31	27	24	21	17	14	11	7	4
15	31	28	25	23	20	17	14	11	8	5	2
0	25	23	20	18	15	13	10	8	5	3	0

T I M E

Table 7. Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 75

MODEL=1

Y(1,1)= 75 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 75
 AEXP= 1 DEXP= 1 KONA= 1 KOND= 1
 A(1)= 0 A(2)=180 D(1)= 0 D(2)=100
 KSTARA=180 KSTOPA= 0 KINCA=-15
 KSTARD= 0 KSTOPD=100 KINCD= 10

Y = 75 + [< .1389+00>*(T - 0) ** 1] + [< -75.00 +00>*(F - 0) ** 1]
 + [< .2778-00>*(T - 0) ** 1] * [(F - 0) ** 1]]

F I L L

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	97	95	92	90	87	85	82	80	77	75
165	98	95	92	89	86	83	80	77	75	72	69
150	96	92	89	86	82	79	76	72	69	66	62
135	94	90	86	82	79	75	71	67	64	60	56
120	92	87	83	79	75	71	67	62	58	54	50
105	90	85	80	76	71	67	62	57	53	48	44
90	87	82	77	72	67	62	57	52	47	42	37
75	85	80	75	69	64	58	53	47	42	37	31
60	83	77	72	66	60	54	48	42	37	31	25
45	81	75	69	62	56	50	44	37	31	25	19
30	79	72	66	59	52	46	39	32	26	19	12
15	77	70	63	56	49	42	35	27	20	13	6
0	75	68	60	53	45	38	30	23	15	8	0

T I M E

Table 8. Y (Payoff) = Second Degree Function of t (time used) and Second Degree Function of f (fraction fill) for k = 25

MODEL=1

Y(1,1)= 25 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 25
 AEXP= 2 DEXP= 2 KONA= 1 KOND= 1
 A(1)= 0 A(2)=180 U(1)= 0 U(2)=100
 KSTARA=180 KSTOPA= 0 KINCA=-15
 KSTARD= 0 KSTOPD=100 KINCD= 10

Y = 25 + [< .2315=02 > * (T - 01 ** 2)] + [< = 25.00=00 > * (F - 01 ** 2)]
 + [< = .1543=02 > * ((T - 01 ** 2) * (F - 01 ** 2))]

F I L L

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	99	97	93	88	81	73	63	52	39	25
165	88	87	85	82	77	71	64	55	45	34	21
150	77	76	75	72	68	62	56	48	39	29	17
135	67	67	65	62	59	54	48	41	33	24	14
120	58	58	56	54	51	47	41	35	28	20	11
105	51	50	49	47	44	40	35	30	24	16	9
90	44	43	42	40	38	34	30	25	20	13	6
75	38	38	37	35	33	30	26	22	16	11	4
60	33	33	32	31	28	26	22	18	14	9	3
45	30	29	29	27	25	23	20	16	12	7	2
30	27	27	26	25	23	20	18	14	10	6	1
15	26	25	25	23	21	19	16	13	9	5	0
0	25	25	24	23	21	19	16	13	9	5	0

Table 9. Y (Payoff) = Second Degree Function of t (time used) and Second Degree Function of f (fraction fill) for k = 75

MODEL=1

Y(1,1)= 75 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 75
 AEXP= 2 DEXP= 2 KONA= 1 KOND= 1
 A(1)= 0 A(2)=180 D(1)= 0 D(2)=100
 KSTARA=180 KSTOPA= 0 KINCA=-15
 KSTARD= 0 KSTOPD=100 KINCD= 10

Y = 75 + [< .7716=03 > * (T - 0) ** 2] + [< - 75.00=00 > * (F - 0) ** 2]
 + [< .1543=02 > * (T - 0) ** 2] * (F - 0) ** 2]]

FILL

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	100	99	98	96	94	91	88	84	80	75
165	96	96	95	93	91	88	84	80	75	69	63
150	92	92	91	89	86	82	78	73	67	60	52
135	89	89	87	85	82	77	72	66	59	51	42
120	86	86	84	81	78	73	67	60	52	43	33
105	84	83	81	78	74	69	63	55	46	37	26
90	81	81	79	76	71	66	59	51	41	31	19
75	79	79	77	73	69	63	55	47	37	26	13
60	78	77	75	72	67	60	53	44	33	22	8
45	77	76	74	70	65	59	51	41	31	18	5
30	76	75	73	69	64	57	49	40	29	16	2
15	75	74	72	68	63	57	48	39	27	15	1
0	75	74	72	68	63	56	48	38	27	14	0

If the policy maker decides that the changes near $T = 0$ and $F = 0$ should be more gradual, then higher exponents could be used. Also the changes might be more gradual for D than for A .

Assume that

AEXP = 3

DEXP = 5

The outputs of this new function for $K = 25$ and $K = 75$ are shown as Tables 10 and 11. Three-dimensional representations of these two models are shown as Figures 16 and 17.

Table 10. Y (Payoff) = Third Degree Function of t (time used) and Fifth Degree Function of f (fraction fill) for $k = 25$

MODEL=1												
Y(1,1)= 25 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 25												
AEXP= 3 DEXP= 5 KONA= 1 KONDA= 1												
A(1)= 0 A(2)=180 D(1)= 0 D(2)=100												
KSTARA=180 KSTOPA= 0 KINCA=-15												
KSTARD= 0 KSTOPD=100 KINCD= 10												
$Y = 25 + [< .1286-04 > * (T - 0) ** 3] + [< -25.00-00 > * (F - 0) ** 5]$ $+ [< -.8573-05 > * [(T - 0) ** 3] * [(F - 0) ** 5]]$												
F I L L												
.0 .10 .20 .30 .40 .50 .60 .70 .80 .90 1.00												
T I M E	180	100	100	100	100	99	98	94	87	75	56	25
	165	83	83	83	83	82	81	78	72	62	45	19
	150	68	68	68	68	68	67	64	59	51	37	14
	135	57	57	57	57	56	55	53	49	42	29	11
	120	47	47	47	47	47	46	44	41	34	24	7
	105	40	40	40	40	40	39	37	34	28	19	5
	90	34	34	34	34	34	33	32	29	24	16	3
	75	30	30	30	30	30	30	28	26	21	14	2
	60	28	28	28	28	28	27	26	23	19	12	1
	45	26	26	26	26	26	25	24	22	18	11	0
	30	25	25	25	25	25	25	23	21	17	10	0
	15	25	25	25	25	25	24	23	21	17	10	0
0	25	25	25	25	25	24	23	21	17	10	0	

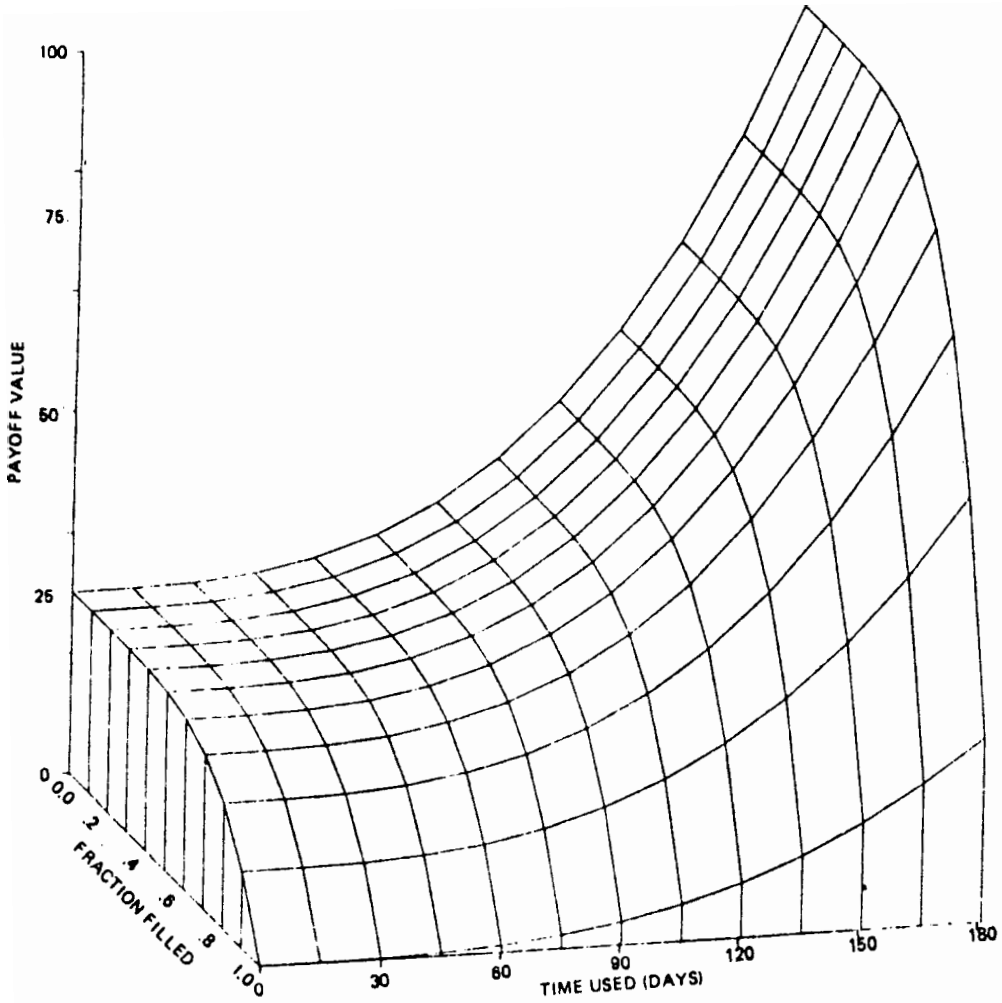


Figure 16. $Y =$ third degree function of t (time used) and fifth degree function of f (fractional fill) for $k = 25$.

Table 11. Y (Payoff) = Third Degree Function of t (time used) and Fifth Degree Function of f (fraction fill) for k = 75

MODEL=1

Y(1,1)= 75 Y(2,1)= 100 Y(1,2)= 0 Y(2,2)= 75
 AEXP= 3 DEXP= 5 KONA= 1 KOND= 1
 A(1)= 0 A(2)=180 D(1)= 0 D(2)=100
 KSTARA=180 KSTOPA= 0 KINCA=-15
 KSTAR0= 0 KSTOPD=100 KINCD= 10

Y = 75 + [< .4287-05 > * (T - 0) * * 3] + [< - .75.00-00 > * (F - 0) * * 5]
 + [< .8573-05 > * ((T - 0) * * 3) * (F - 0) * * 5]]

F I L L

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	100	100	100	100	99	98	96	92	85	75
165	94	94	94	94	94	93	91	88	82	73	58
150	89	89	89	89	89	88	86	82	74	62	43
135	86	86	86	85	85	84	81	76	68	54	32
120	82	82	82	82	82	81	78	72	63	47	22
105	80	80	80	80	79	78	75	69	59	42	15
90	78	78	78	78	77	76	73	67	56	38	9
75	77	77	77	77	76	75	71	65	53	35	5
60	76	76	76	76	75	74	70	64	52	33	3
45	75	75	75	75	75	73	70	63	51	32	1
30	75	75	75	75	74	73	69	63	51	31	0
15	75	75	75	75	74	73	69	62	50	31	0
0	75	75	75	75	74	73	69	62	50	31	0

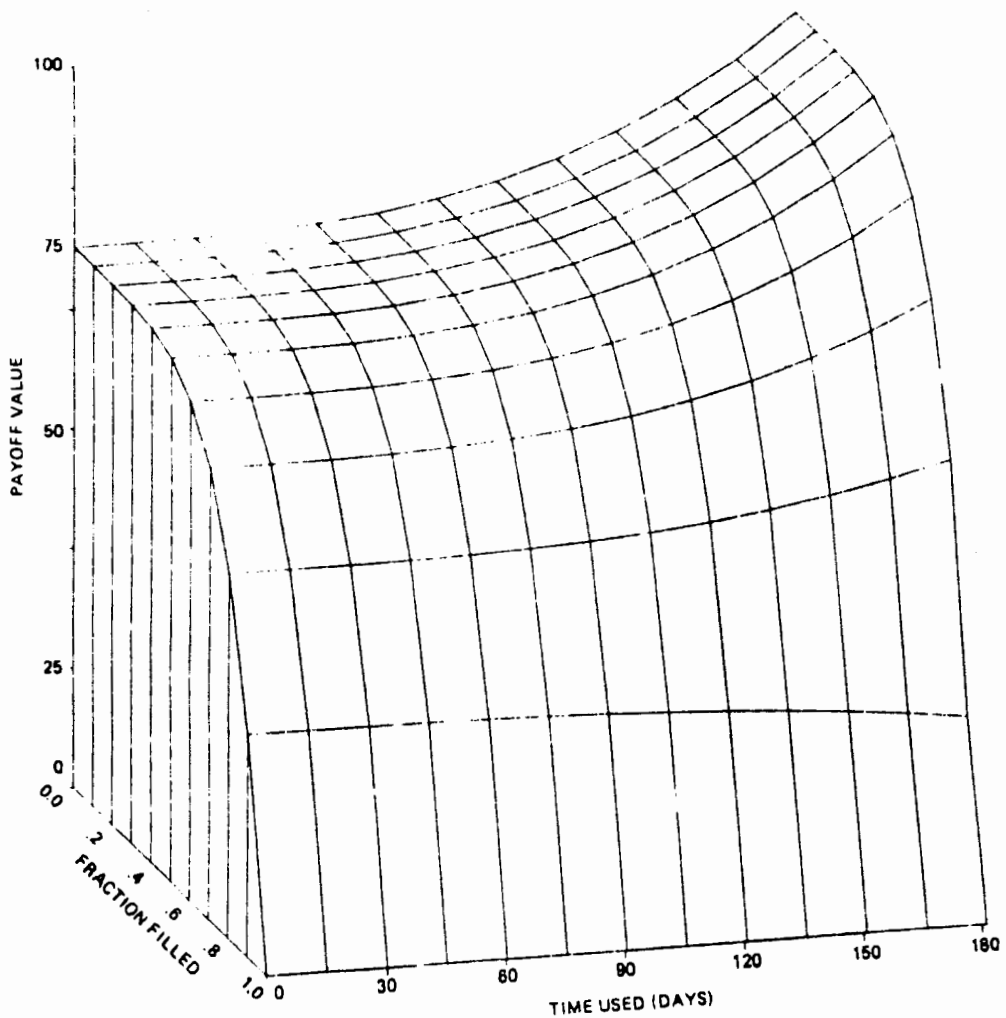


Figure 17. $Y =$ third degree function of t (time used) and fifth degree function of f (fractional fill) for $k = 75$.

Model 2

The second general model allows for expressing the composite value Y as a function of any two variables A and D using a general polynomial form as in Model 1. However, this model allows for more complex control of the slopes so that policy expressions that reflect statements about maximum or minimum values can be easily specified. Model 2 is developed in Appendix F.

Model 2 is expressed as

$$\begin{aligned}
 Y &= B(1) + B(2) * (A - A(KONA)) ** AEXP \\
 &\quad + B(3) * (D - D(KOND)) ** (DEXP - 1) \\
 &\quad + B(4) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** DEXP) \\
 &\quad + B(5) * ((A - A(KONA)) ** AEXP) * ((D - D(KOND)) ** (DEXP - 1)) \\
 &\quad + B(6) * (D - D(KOND)) ** DEXP
 \end{aligned}$$

Where

$$\begin{aligned}
 B(1) &= Y(KONA, KOND) \\
 B(2) &= (Y(KONACH, KOND) - Y(KONA, KOND)) / ((A(KONACH) - A(KONA)) ** AEXP)
 \end{aligned}$$

$$\begin{aligned}
B(3) &= (DEXP * (Y(KONA, KONDCH) - Y(KONA, KOND)))/((D(KONDCH) - D(KOND)) \\
&\quad ** (DEXP - 1)) \\
B(4) &= (Y(KONACH, KONDCH) - Y(KONACH, KOND) + ((DEXP - 1) * (Y(KONA, \\
&\quad KONDCH) - Y(KONA, KOND)))/(((A(KONACH) - A(KONA)) ** AEXP) * \\
&\quad ((D(KONDCH) - D(KOND)) ** DEXP)) \\
B(5) &= (-B(3))/(A(KONACH) - A(KONA)) ** AEXP \\
B(6) &= ((-B(3)) * (DEXP - 1))/(DEXP * (D(KONDCH) - D(KOND)))
\end{aligned}$$

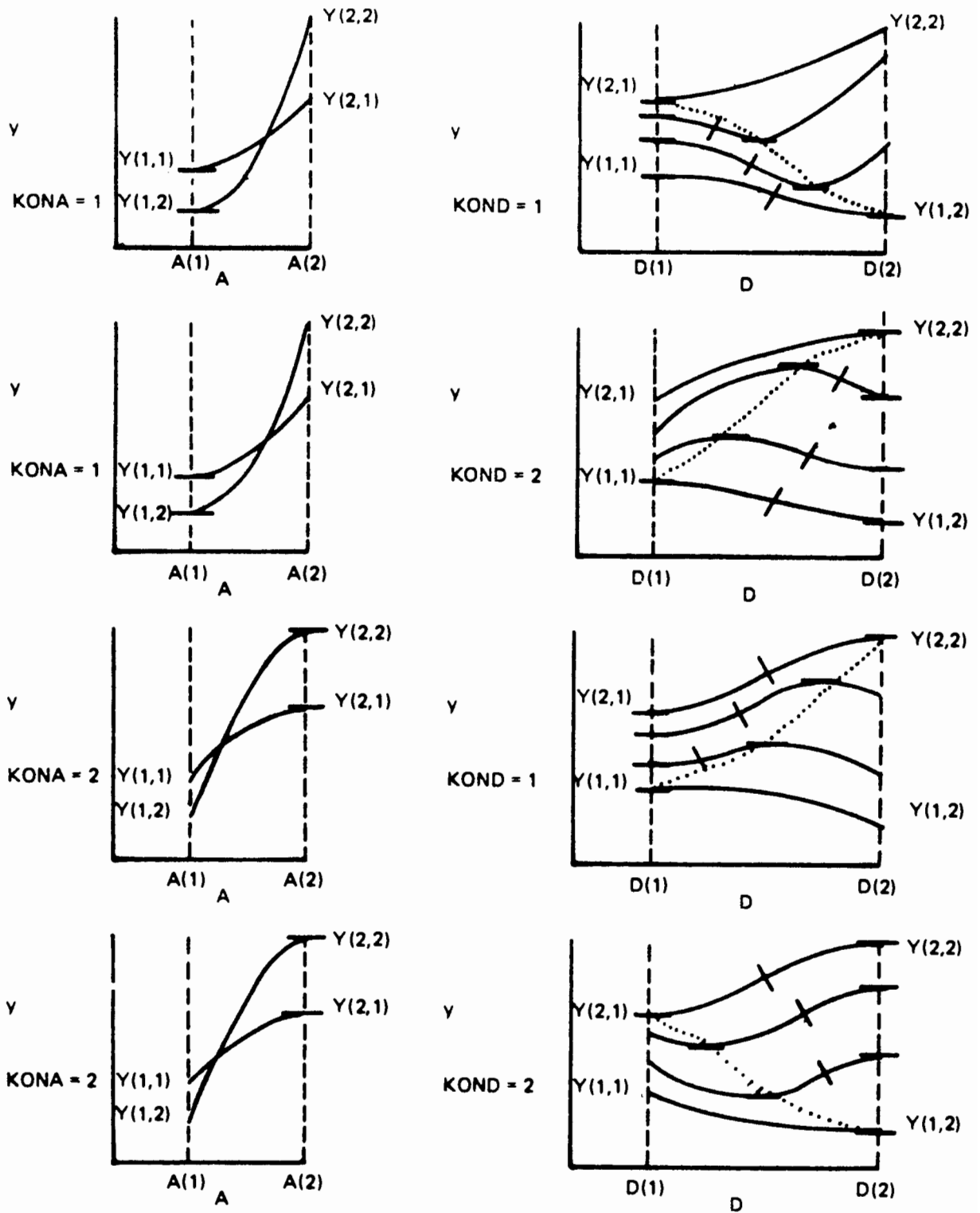
This model has the following characteristics, the first three being the same as for Model 1:

1. For every value of D the A-slope is zero at A(KONA) and no other values of A.
2. For every value of A the D-slope is zero at D(KOND).
3. For any value of AEXP and for all values of D the variable A and A-slope are either always increasing or always decreasing between A(1) and A(2). Note as before that higher values of AEXP result in slower changes in Y as A changes near A(KONA) and faster changes in Y as A changes near A(KONACH).
4. For any value of $DEXP \geq 3$, the D-slope variable is zero at D(KONDCH) and A(KONA).
5. For any value of $DEXP \geq 3$, and for A(KONA) there is only one inflection point for D-slope between the zero slope at D(KOND) and the zero slope at D(KONDCH).
6. For any value of $DEXP \geq 3$ and for A(KONACH) the variable D and the D-Slope are either always increasing or always decreasing between D(1) and D(2). Note that higher values of DEXP result in slower changes in Y as D changes near D(KOND) and faster changes near D(KONDCH).
7. The previous restrictions result in the movement of D-slope = 0 from A(KONA), D(KONDCH) to A(KONACH), D(KOND). This allows for policy specification requiring ridges or valleys indicated by dotted lines on the right side of Figure 18. These sketches indicate how Model 2 might appear with selected settings of Y's and D's for all combinations of KONA and KOND.

The general form of Model 2 will now be used to represent Example 1 above where Y is a function of Aptitude, A, and Job Difficulty, D.

Here let variable A correspond to A (aptitude), and variable D correspond to D (difficulty). Now use Model 2 to create the required policy specification.

A(1)	=	40, the smaller control value for A (aptitude)
A(2)	=	95, the larger control value for A (aptitude)
D(1)	=	40, the smaller control value for D (difficulty)
D(2)	=	100, the larger control value for D (difficulty)
Y(1,1)	=	15, Y value at A = 40, D = 40
Y(2,1)	=	35, Y value at A = 95, D = 40
Y(1,2)	=	-250, Y value at A = 40, D = 100, and determined by experience with different values to obtain policy specification 3 for the example.
Y(2,2)	=	100, Y value at A = 95, D = 100
AEXP	=	1, change in Y is constant as A changes
DEXP	=	3, value of DEXP that gives control of maximum values at desired places. Higher values can be tried to observe Y value characteristics.
KONA	=	2, provides for formation of a ridge of maximum Y values from A = 40, D = 40 to A = 95, D = 100



Note: Horizontal Dark Lines (—) indicate slope = 0.
 Slanting Dark lines (/) indicate inflection points
 Dotted Lines (...) indicate ridges or valleys.

Figure 18. Y (payoff) = function of A and D . Examples of parameter settings for Model 2.

$$\begin{aligned} \text{KOND} &= 1 \text{ which provides gradual increase of } Y \text{ as difficulty changes near } D = 40 \text{ for all values of aptitude, allows for rapid decrease of } Y \text{ values as difficulty changes after maximum } Y \text{ values} \\ \text{KONACH} &= 3 - \text{KONA} = 1 \\ \text{DONDCH} &= 3 - \text{KOND} = 2 \end{aligned}$$

Substitution in Model 2 gives

$$\begin{aligned} \text{B(1)} &= Y(\text{KONA}, \text{KOND}) = Y(2,1) = 35 \\ \text{B(2)} &= (Y(\text{KONACH}, \text{KOND}) - Y(\text{KONA}, \text{KOND})) / ((A(\text{KONACH}) - A(\text{KONA})) ** \text{AEXP}) \\ &= (Y(1,1) - Y(2,1)) / ((A(1) - A(2)) ** \text{AEXP}) \\ &= (15 - 35) / (40 - 95) ** 1 \\ &= -20 / -55 = 20 / 55 = .3636 \\ \text{B(3)} &= (\text{DEXP} * (Y(\text{KONA}, \text{KONDCH}) - Y(\text{KONA}, \text{KOND}))) / ((D(\text{KONDCH}) - D(\text{KOND})) ** (\text{DEXP} - 1)) \\ &= (3 * (Y(2,2) - Y(2,1))) / (D(2) - D(1)) ** 2 \\ &= (3 * (100 - 35)) / (100 - 40) ** 2 \\ &= (3 * 65) / 3600 = 65 / 1200 = .05417 \\ \text{B(4)} &= (Y(\text{KONACH}, \text{KONDCH}) - Y(\text{KONACH}, \text{KOND}) + ((\text{DEXP} - 1) * (Y(\text{KONA}, \text{KONDCH}) - Y(\text{KONA}, \text{KOND})))) / (((A(\text{KONACH}) - A(\text{KONA})) ** \text{AEXP}) * ((D(\text{KONDCH}) - D(\text{KOND})) ** \text{DEXP})) \\ &= (Y(1,2) - Y(1,1)) + ((2 * (Y(2,2) - Y(2,1)))) / (((A(1) - A(2)) ** 1) * ((D(2) - D(1)) ** 3)) \\ &= ((-250 - 15) + 2 * (100 - 35)) / ((40 - 95) * (100 - 40) ** 3) \\ &= (-265 + 2 * 65) / (-55) * (60) ** 3 \\ &= (-265 + 130) / (-55 * 216 * 1000) \\ &= -135 / (-55 * 216 * 1000) = .00001136 \\ \text{B(5)} &= (-\text{B(3)}) / (A(\text{KONACH}) - A(\text{KONA})) ** \text{AEXP} \\ &= (-65 / 1200) / (A(1) - A(2)) ** 1 \\ &= (-65 / 1200) / (40 - 95) = 65 / (1200 * 55) \\ &= .0009848 \\ \text{B(6)} &= ((-\text{B(3)}) * (\text{DEXP} - 1)) / (\text{DEXP} * (D(\text{KONDCH}) - D(\text{KOND}))) \\ &= ((-65 / 1200) * 2) / (3 * (100 - 40)) \\ &= (-130 / 1200) / (3 * (60)) \\ &= -.0006019 \end{aligned}$$

The equation is the same as shown in Example 1 given earlier.

$$\begin{aligned} Y &= 35 + .3636 (A - 95) + .05417 (D - 40)^2 \\ &\quad + .00001136 (A - 95)(D - 40)^3 \\ &\quad + .0009848 (A - 95)(D - 40)^2 \\ &\quad - .0006019 (D - 40)^3 \end{aligned}$$