CREATING MATHEMATICAL MODELS OF JUDGMENT PROCESSES: FROM POLICY-CAPTURING TO POLICY-SPECIFYING

I. INTRODUCTION

Following feasibility studies (Hawkins, Crow, & Haltman, 1974) and demonstrations (Ward & Haltman, 1975) a computer-based system was being developed for selecting and classifying personnel who are considering enlisting in the Air Force. This system is a part of the Air Force's Advanced Personnel Data System Procurement Management Information System (APDS-PROMIS). Within this system there is a need for numerical values that will indicate the "value" or "payoff" to the Air Force or recruiting a particular person for a particular job at a specific time. The value to be used is sometimes determined by judgment of one or more policy makers. Frequently this value is obtained by combining several different types of information into a weighted composite to produce a numerical indicator of the policy maker's judgment of "value."

One method of weighting is to have the policy maker explicitly provide the numerical weights to be used with the different types of information to form the composite (explicit-weighting). Explicit-weighting is satisfactory in some situations. It is usually difficult, however, to choose the proper multiplier values to form the composite values that adequately express the worth of a person on a job.

The difficulties encountered with explicit-weighting have led to a second method – policy-capturing – which involves implicit determination of the numerical weights. In the policy-capturing process the policy maker observes various decision situations and assigns a number to reflect the "value" of each situation. For example, a policy maker may be presented a series of information profiles each of which reflects important data about an Air Force applicant, such as aptitude test scores, difficulty of the job being considered, applicant preferences, etc. The policy maker assigns a number to each profile, which reflects the value to the Air Force of assigning the applicant to a particular job. Then the weights are computed – by least squares regression – that best predict the judged values from variables derived from the information available about each decision situation. Some examples of policy-capturing applications have been described in the following publications: Black (1973); Christal (1968a, 1968b); Gott (1974); Gooch (1972); Jones, Mannis, Martin, Summers, and Wagner (1976); Koplyay (1970); Koplyay, Albert, and Black (1976); Mullins and Usdin (1970); Ward and Davis (1963).

During the early stages of development of the APDS-PROMIS system many discussions took place concerning various approaches to obtaining an expression of value for the different person-job assignments. It seemed appropriate to create the values of the assignments by means of the policy-capturing process. However, there was considerable hesitancy by everyone concerned to launch into policy-capturing. Some other approach seemed to be required.

It is difficult to identify all of the reasons for not carrying through the policy-capturing process. However, the major difficulties seemed to be related to the existence of two different types of information to be weighted into the value composite. The first type might be called *management-related*, such as "filling of quotas" and "maintaining minority balance," and the second type, *quality-of-assignments-related*, such as "matching a person to a job in which he will perform well and be satisfied." The two types of information contributed to another problem in policy-capturing – "who will be the panel of policy makers?" There might be judges who can adequately combine the management-related information and there might be judges who can handle the quality-of-assignments-related information. But it was felt that it would be difficult to identify policy makers who could appropriately combine both types of variables into an acceptable policy through the policy-capturing process. Furthermore, it seemed likely that a mixed panel of policy makers (management-related vs. quality-of-assignments-related) would not yield an acceptable model through policy-capturing. And failure to arrive at a policy model could strain relations among the policy makers.

Time was getting short. A value generator was needed. It was decided that a starting policy model should be created reflecting as well as possible the "expressions of policy" that had emerged through the numerous discussions among personnel managers and researchers. Output values from this starting model would be displayed and used in a demonstration of the PROMIS assignment procedure. Both personnel managers and researchers could examine the generated values and provide comments to the model makers for revising the model. This strategy led to the development of another implicit-weighting approach – policy-specifying. This third method provides another way of reflecting a policy maker's value judgments in a mathematical model without requiring explicit-weighting and without the more lengthy policy-capturing process.

The process of policy-specifying requires a translation into a mathematical model of the policy makers' general statements about the way information is combined to generate values. After a model is created and the results are displayed to the policy makers, a new model is evolved to reflect the policy more precisely. The process of creating new models with new properties continues until the outputs are acceptable to the policy makers.

Repeatedly creating new models for the generation of the PROMIS payoffs was time-consuming and the long cycle time required to display the output from a new model threatened an unacceptable delay in implementing the APDS-PROMIS system.

In order to reduce the time required to create new models for examination by policy makers, it was decided to develop a model generating system. This model generator is controlled by parameter settings through an interactive computer program. It allows the model developer to greatly reduce the time required to bring models into alignment with desired policy.

The power of this model generator was realized during the last two weeks before nationwide operational implementation of APDS-PROMIS. Policy makers took a last minute closer look at the existing (soon to be operational) payoff generator. They were not happy. Using a remote terminal connected by phone from the policy maker's office at Randolph AFB to the AFHRL UNIVAC 1108 Computer at Lackland AFB new models were quickly examined and tested in the operational system. An acceptable payoff generator was created and APDS-PROMIS went operational on schedule.

The general models developed in this report for policy-specifying in the personnel assignment problem can be applied to many other situations by varying the parameters of the models to obtain the desired characteristics. When these model forms are not applicable, the appropriate models can be developed by imposing the restrictions systematically as shown in Appendixes A through G of this report and in *Introduction to Linear Models* (Ward & Jennings, 1973) and *Applied Multiple Linear Regression* (Bottenberg & Ward, 1963).

The remainder of this report focuses in Section II on the two implicit-weighting methods mentioned above – policy-capturing and policy-specifying – and their combination referred to as policy-development. Section III contains several specific examples of policy-specifying that arose during the development of the person-job-match component of APDS-PROMIS. Section IV contains a description of the model generator and its application to the specific examples in Section III. The detailed developments of the models and the FORTRAN program for the model generator are included as appendixes.

II. IMPLICIT WEIGHTING IN POLICY DEVELOPMENT

The weights derived from a policy-development process can be viewed on an implicit-weighting continuum. On one end of the continuum are the implicit weights derived from the policy-capturing process, and at the other extreme are the implicit weights derived from the policy-specifying process. This section focuses first on the implicit weights derived from policy-capturing and policy-specifying. This is followed by a discussion of the implicit weights from general policy-development which combines both capturing and specifying.

Policy-Capturing Weights

Extensive discussions of the policy-capturing process are available in the references previously cited. The focus here is on the regression weights obtained from policy-capturing. The implicit weights obtained from policy-capturing are obtained by solving for coefficients that when used to form a composite of predictor information will best predict the judgments.

Specifically, let

Y = a vector of judged values of dimension n. These n judgments are obtained from a policy maker, who examines each decision situation and assigns a value to be associated with the situation.

U = the unit vector of dimension n, with all elements equal 1.

$X^{(j)}$ = the jth predictor vector, of dimension n generated from the information associated with the decision situations. $j = 1, \ldots, k$. Assume that $n > k + 1$.

E = the error vector of dimension n.

a_j = the unknown weights associated with $X^{(j)}$ to be implicitly determined to minimize the error sum of squares. $j = 1, \ldots, k$.

a_o = the unknown weight associated with U.

Then the prediction model may be written

$$Y = a_o U + a_1 X^{(1)} + a_2 X^{(2)} + \ldots + a_j X^{(j)} + \ldots + a_k X^{(k)} + E^{(1)} \quad (1)$$

and the least squares weights can be used to predict the Y values for new situations.

While the policy-capturing process has been quite useful for modeling judgments, another procedure for obtaining weights may have advantages in some situations.

Policy-Specifying Weights

Policy-capturing requires a set of judgments (Y values) associated with n decision situations to obtain the implicit weights. However, in the policy-specifying process the weights are determined without empirically obtained judgments (Y values) by stating desired properties of and relations among the predicted values in sufficient detail that the numerical weights become known.

Specifically let

b_j = the unknown weights to be determined by policy-specifying (corresponding to a_j in policy-capturing above). $j = 1, \ldots, k$.

b_o = an unknown constant (corresponding to a_o)

x_j = variables corresponding to the predictor vectors above. These are not *vectors* of data but are variables which when given a set of weights b_j and b_o and a set of values for x_j will yield a composite value y.

Then we have the starting function

$$y = b_o + b_1 x_1 + b_2 x_2 + \ldots + b_j x_j + \ldots + b_k x_k \quad (2)$$

Prior to the policy-specifying process the range of values for $x_1, x_2, \ldots, x_k$ are known but the b_j and b_o values are not known. Policy-specifying proceeds by stating restrictive relations among the predicted values for various values of x_j. These policy statements result in restrictions on the values of b_j and b_o so that the numerical values of the weights can be determined. Specification is completed when k + 1 independent restrictions are imposed. Once the values of b_j and b_o are known then predicted values, y, can be calculated for any values x_j.

This process of policy-specifying is the same as that required for imposing restrictions on linear statistical models to test hypotheses. The procedures described by Bottenberg and Ward (1963) and Ward and Jennings (1973) can be directly applied in determining the weights implied by the policy-specifying process.

This report focuses on specific examples of policy-specifying and general forms which might be applicable to new situations.

Policy-Development Weights

Policy-capturing and policy-specifying can be combined to form a general process of policy-development. A particular decision maker may start by specifying several properties about relations among the predicted values in the function (2). Whereas policy-specifying resulted in k + 1 restrictions on the k + 1 weights, b_j and b_o, the expression of desired properties may result in only $r < k + 1$ restrictions on the b_j and b_o values.

Then imposing these r restrictions on the starting model (2) results in a restricted model

$$y_r = c_o + c_1 z_1 + c_2 z_2 + \ldots + c_j z_j + \ldots + c_{k-r} z_{k-r} \quad (3)$$

where

z_i = new variables resulting from imposing the r restrictions.

Each z_i variable is a linear combination of the x_i variables. Now since there are still k + 1 - r unknown weights c_j and c_o to be computed it would be possible to use policy-capturing to find the c_j values. The decision maker could provide, for each of n $[n > (k + 1 - r)]$ decision situations, y_i $(i = 1, \ldots, n)$ values associated with various profiles of information about the different situations. Then the least squares values of c_j can be computed for the model

$$Y = c_o U + c_1 Z^{(1)} + c_2 Z^{(2)} + \ldots + c_j Z^{(j)} + \ldots + c_{k-r} Z^{(k-r)} + E^{(2)} \quad (4)$$

where

Y = a vector of judged values of dimension n.

$Z^{(j)}$ = the jth predictor vector, of dimension n formed as linear combinations of the predictor vectors $X^{(j)}$ generated from information associated with the decision situations.

Having computed the least squares values for c_j and c_o the weighting system now produces values that both reflect the policy restrictions imposed by the policy-specifying process and the best fit to the empirical judgments.

The remainder of this report will concentrate on policy-specifying procedures. The next section will provide several examples of the use of policy-specifying. This will be followed by generalizations which are applicable to new situations.

III. POLICY-SPECIFYING EXAMPLES

This section contains several examples of policy-specifying that arose during the development of the person-job-match component of APDS-PROMIS. Focus in these examples is on description of the policy and the model that results from translating the natural language policy statements into mathematical form. Model specifying details which are required to develop the models are presented in the appendixes. The mathematical restrictions, imposed in the appendixes to create the various models, are selected to both approximate the policy statements and to allow for easy generation and control of the models.

Example 1: Value to Air Force as Function of Aptitude and Job Difficulty

The most important example arising in PROMIS is the expression of policy about the "value to the Air Force" of assigning a particular person to a particular job. While there are several variables (or components) that contribute to this expression of worth, the most important component involves the expression of value of the person-job assignment as a function of only two basic properties – aptitude of the person and difficulty of the job.

The policy maker indicated the following desired properties of his values:

1. The range of composite numbers y to express "value" would be from 0 to 100.

2. A value of 100 would be assigned when a person with maximum aptitude (a = 95) is assigned to a job of maximum difficulty (d = 100).

3. Values of 0 would be assigned when a person's aptitude is about 15 or 20 points below the difficulty measure of the job.

4. A value of 15 would be assigned when a person with minimum aptitude (a = 40) is assigned to a job of minimum difficulty (d = 40).

5. A value of 35 would be assigned when a person with maximum aptitude (a = 95) is assigned to a job of minimum difficulty (d = 40).

6. The values for a person with maximum aptitude (a = 95) will start at y = 35 when a = 40 and increase gradually, reaching the maximum value y = 100 at d = 100. This policy statement is sketched in Figure 1.

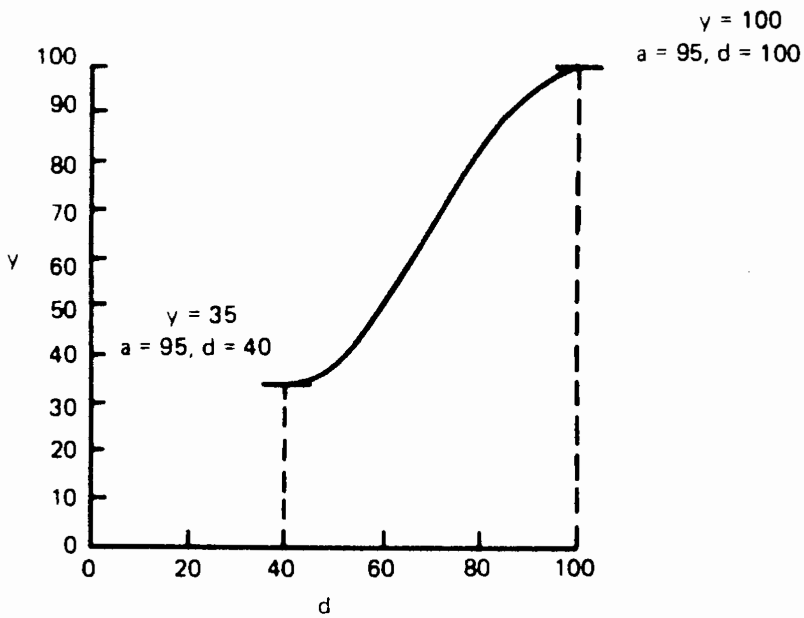

Figure 1. **Y (payoff) = function of d at a = 95.**

7. A person of minimum aptitude (a = 40) will have a maximum value (y = 15) when assigned to a minimum difficulty job (d = 40). The values for this person will start at y = 15 when a = 40 and decrease gradually to y = 0 when the job difficulty is about 60. This can be sketched as shown in Figure 2. The combination of policies 6 and 7 are shown in Figure 3.

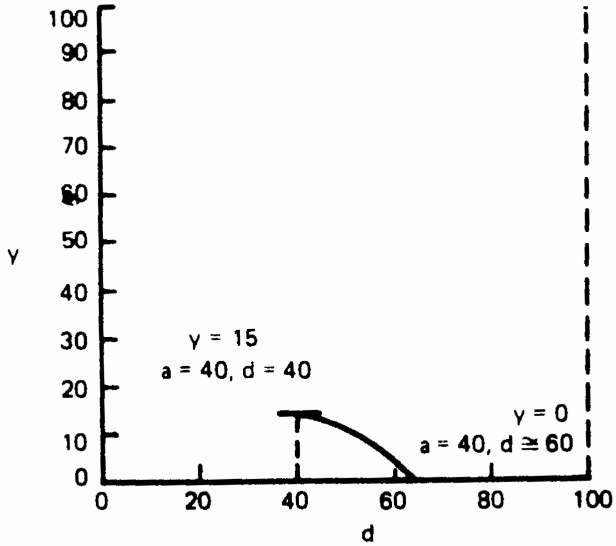

Figure 2. Y (payoff) = function of d at a = 40.

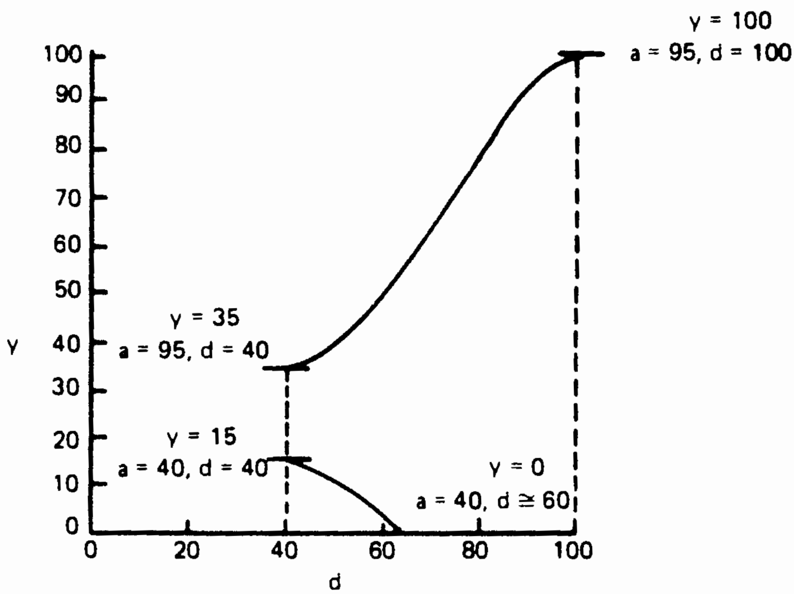

Figure 3. Y (payoff) = function of d at a = 40 and a = 95.

8. Persons between the extreme aptitudes ($40 \leqslant a \leqslant 95$) will have their maximum values about when the aptitude value is approximately equal to or slightly greater than the difficulty measure. This policy can be combined with statements 6 and 7 to give the following sketch shown in Figure 4.

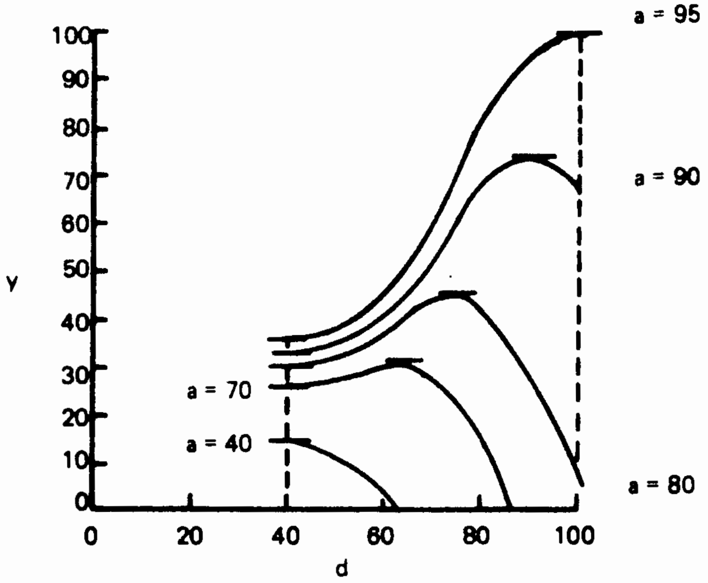

Figure 4. **Y (payoff) = function of d at a = 40, a = 70, a = 80, a = 90 and a = 95.**

9. The policy maker stated that for a job of minimum difficulty (d = 40) the amount of change in value per unit change in aptitude would be constant and the values should increase only slightly. As a moves from 40 to 95, y changes from 15 to 35.

10. As the job difficulty increases, the amount of change in value per unit change in aptitude increases rapidly. Statements 9 and 10 are sketched as shown in Figure 5.

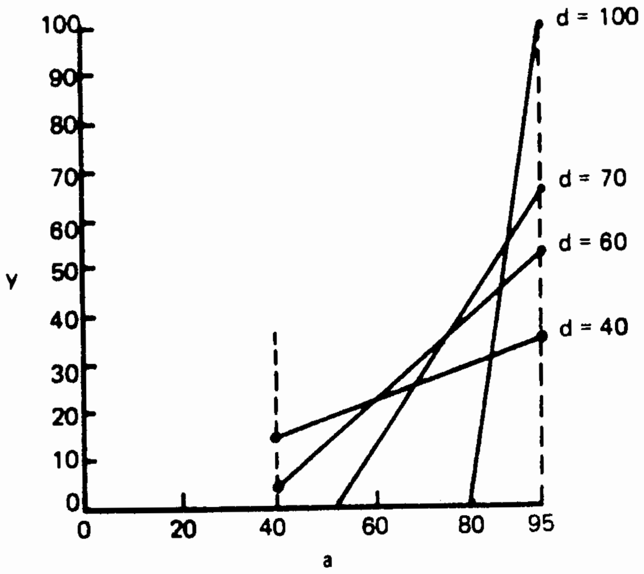

Figure 5. **Y (payoff) = function of a at d = 40, d = 60, d = 70 and d = 100.**

Following the procedures detailed in Appendix A, the policy specified model becomes

$$\begin{aligned} y = 35 \; & + .3636\,(a-95) + .05417\,(d-40)^2 \\ & + .00001136\,(a-95)\,(d-40)^3 \\ & + .0009848\,(a-95)\,(d-40)^2 \\ & - .0006019\,(d-40)^3 \end{aligned}$$

The highest power for aptitude (variable a) was selected as 1 since statement 9 indicates a constant change in value (y) per unit change in aptitude (a). The highest power of difficulty (variable d) was selected as small as possible (3) while reflecting statements 6 and 7. Higher powers of d could be used and presented to policy makers if power 3 does not produce desired outcomes. The product between aptitude (a) and difficulty (d) reflects interactions between a and d as implied in statements 6 and 7, and in statements 9 and 10.

Table 1 contains selected values generated from the above policy-specified model. Observe that values outside the critical range are shown to illustrate that the model has the desired properties. Only the positive values generated by this model are used. The eligibility tests applied prior to use of this model eliminate aptitude-difficulty combinations that produce negative payoff values. Observe that all slopes are zero at d (difficulty) = 40 and that the ridge of maximum values moves from 15 at a = 40, d = 40 to 100 at a = 95, d = 100. Figure 6 is a three-dimensional representation of the model.

Table 1. **Y (Payoff) = Function of a (aptitude) and d (difficulty)**

```
Y =    35 + [< .3636+00>*(A -   95)**  1] + [< .5417-01>*(D -   40)** 2]
          + [< .1136-04>*[(A -   95)**  1]*[(D -   40)** 3]]
          + [< .9848-03>*[(A -   95)**  1]*[(D -   40)** 2]]
          + [<-.6019-03>*(D -   40)**  3]
```

		DIF														
		35	40	45	50	55	60	65	70	75	80	85	90	95	100	105
	100	38	37	38	42	48	56	65	75	86	96	107	116	125	132	137
	95	36	35	36	40	45	52	59	67	76	83	90	95	99	100	99
	90	34	33	34	37	42	48	54	60	65	70	73	74	73	68	60
	85	33	31	32	35	39	43	48	52	55	56	56	53	46	36	22
	80	31	30	30	33	36	39	42	44	45	43	39	31	20	5	-16
	75	29	28	28	30	33	35	36	36	34	30	22	10	-6	-27	-54
A	70	27	26	27	28	30	31	31	29	24	16	5	-11	-32	-59	-93
P	65	25	24	25	26	26	26	25	21	14	3	-12	-32	-58	-91	-131
T	60	23	22	23	23	23	22	19	13	4	-10	-29	-53	-84	-123	-169
	55	21	20	21	21	20	18	13	5	-7	-24	-46	-75	-111	-155	-207
	50	19	19	19	19	17	14	7	-3	-17	-37	-63	-96	-137	-186	-245
	45	17	17	17	16	14	9	2	-10	-27	-50	-80	-117	-163	-218	-284
	40	15	15	15	14	11	5	-4	-18	-38	-64	-97	-138	-189	-250	-322
	35	13	13	13	11	8	1	-10	-26	-48	-77	-114	-160	-215	-282	-360

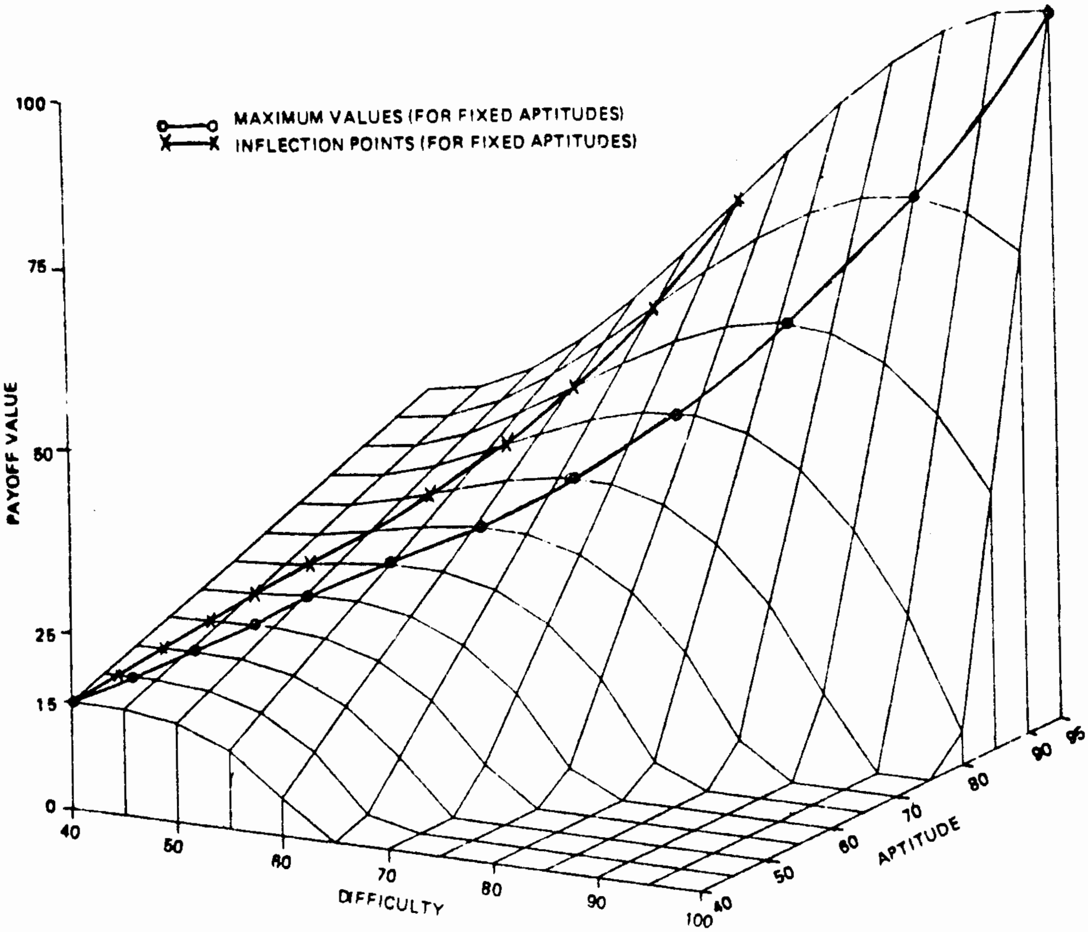

Figure 6. **Aptitude-Difficulty component.**

Example 2: Value to Air Force as Function of Time Used and Fraction of Fill

Another component that contributes to overall value of assigning a particular person to a job is related to the particular job's importance, the time used, and the fraction of the quota already filled.

It is assumed in this example that when a job quota becomes open there are 180 days left to fill the quota.

The policy maker stated the following characteristics of his values:

1. The range of composite numbers y to express "value" is from 0 to 100.

2. There should be an importance number, k, (between 0 and 100) that can be used to emphasize certain jobs more strongly than others independent of the time used and fraction of fill.

3. A value of y close to 100 would be assigned for a completely unfilled quota with almost no time left (near time, t = 180 days and fraction of quota filled, f ≃ .0).

4. A value close to 0 would be assigned for an almost completely filled job with maximum time left (near t = 0 days, f = 1.0).

5. The importance number, k, associated with the job would be the value of y when time used, t = 0 days and fraction of quota filled, f = .0.

6. The value of y approaches k, the job-importance number, when the quota is close to being filled with almost no time left (near t = 180 and f = 1.0).

7. For any fraction of fill, the amount of change in value per unit change in time is constant; however, the constant change might be different for different values of fill.

8. For any time, the amount of change in value per unit change in fraction fill is constant; however, the constant change might be different for different values of time.

Translating these policy statements into a model (details given in Appnedix B) yields

$$y = k + \frac{(100 - k)t}{180} + (-k)f + \frac{(2k - 100)}{180}(tf) \qquad (2)$$

Notice that the highest power for both t and f is one. This reflects statements 7 and 8. Also the product between t and f reflects the interactions implied in statements 7 and 8.

For example, consider a low priority job with k = 25.

Then

$$y = 25 + \frac{75}{180}t - 25f - \frac{50}{180}(tf)$$

$$= 25 + .4167(t) - 25f - .2778(tf)$$

Two sketches of this policy are shown in Figures 7 and 8.

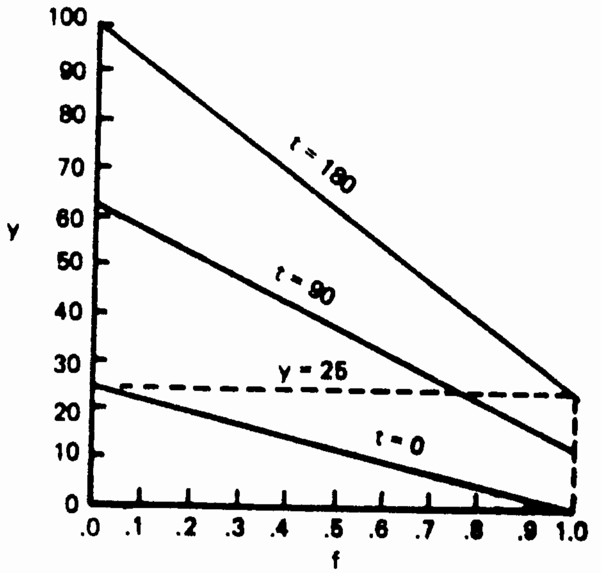

Figure 7. Y (payoff) = function of f for k = 25 at t = 0, t = 90, t = 180.

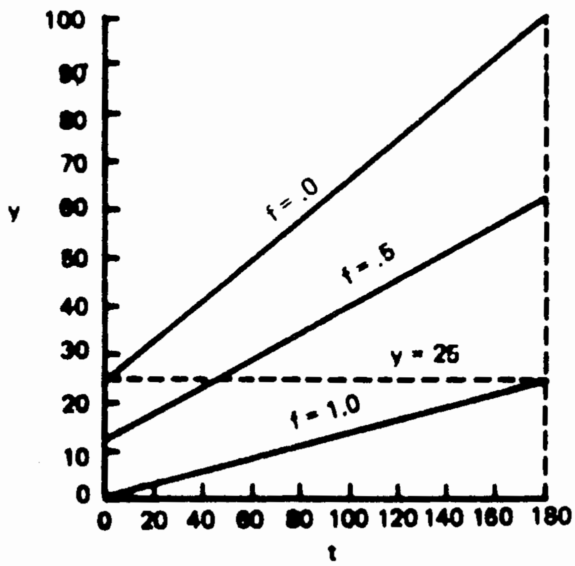

Figure 8. **Y (payoff) = function of t for k = 25 at f = .0, f = .5, f = 1.0.**

Values of y for selected combinations of t and f are shown in Table 2.

Table 2. **Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 25**

```
Y =  25 + ((  .4167+00)*(T -   0)** 1) + ((- 25.00+00)*(F -   0)** 1).
        + ((-.2778-00)*((T -   0)** 1)*((F -   0)** 1))
```

FILL

TIME	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	93	85	78	70	63	55	48	40	33	25
165	94	87	80	73	65	58	51	44	37	30	23
150	87	81	74	68	61	54	48	41	34	28	21
135	81	75	69	63	56	50	44	37	31	25	19
120	75	69	63	58	52	46	40	34	28	22	17
105	69	63	58	52	47	42	36	31	25	20	15
90	62	58	53	48	43	38	33	28	23	18	13
75	56	52	47	43	38	33	29	24	20	15	10
60	50	46	42	38	33	29	25	21	17	12	8
45	44	40	36	33	29	25	21	17	14	10	6
30	37	34	31	27	24	21	17	14	11	7	4
15	31	28	25	23	20	17	14	11	8	5	2
0	25	23	20	18	15	13	10	8	5	3	0

A high priority job, for example k = 75 would give

$$y = 75 + \frac{25}{180}t - 75f + \frac{50}{180}(tf)$$

$$= 75 + .1389(t) - 75(f) + .2778(tf)$$

Two sketches of the high priority policy are shown in Figures 9 and 10.

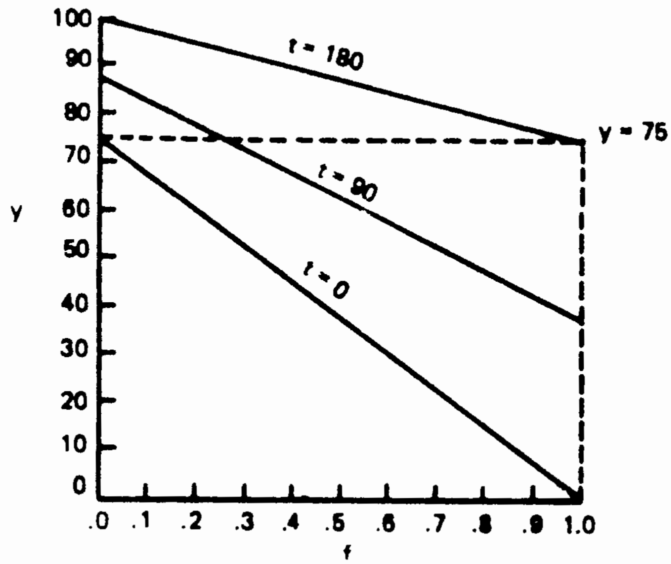

Figure 9. **Y (payoff) = function of f for k =75 at t = 0, t = 90, t = 180.**

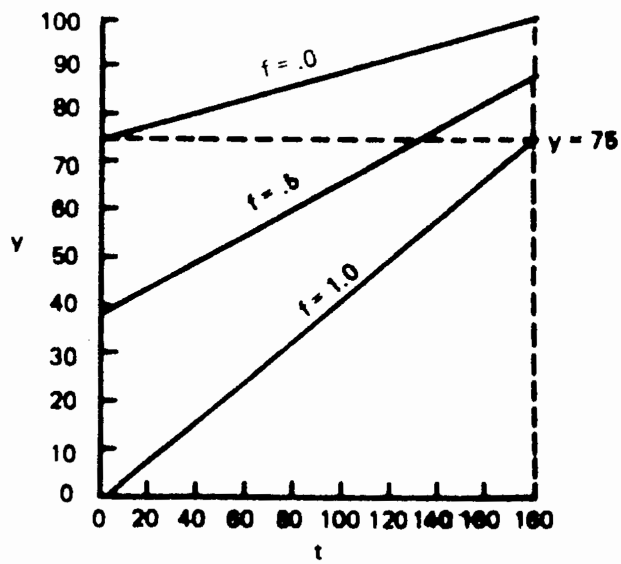

Figure 10. **Y (payoff) = function of t for k = 75 at f = .0, f = .5, f = 1.0.**

Values of y for selected combinations of t and f are shown in Table 3.

Table 3. **Y (Payoff) = Function of t (time used) and f (fraction fill) for k = 75**

```
Y =    75 + ((.1389+00)*(T -     0))**  1 1 + ((- 75.00+00)*(F -     0))**  1 1
          + ((.2773-00)*(T -     0))**  1.1 (F -     0))**  1.1]
```

FILL

TIME	0	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
180	100	97	95	92	90	87	85	82	80	77	75
165	98	95	92	89	86	83	80	77	75	72	69
150	96	92	89	86	82	79	76	72	69	66	62
135	94	90	86	82	79	75	71	67	64	60	56
120	92	87	83	79	75	71	67	62	58	54	50
105	90	85	80	76	71	67	62	57	53	48	44
90	87	82	77	72	67	62	57	52	47	42	37
75	85	80	75	69	64	58	53	47	42	37	31
60	83	77	72	66	60	54	48	42	37	31	25
45	81	75	69	62	56	50	44	37	31	25	19
30	79	72	66	59	52	46	39	32	26	19	12
15	77	70	63	56	49	42	35	27	20	13	6
0	75	68	60	53	45	38	30	23	15	8	0

Example 3: Value to Air Force as Function of Fill and Known Goal

Another type of value component relates to the fraction of quota fill compared to a specific goal. This policy could be developed to reflect the value of an assignment as it relates to the fraction of quota filled by a minority group compared to a desired fraction.

The policy specified in this case involves the following general characteristics:

1. The range of values for y is 0 to 100.
2. When the fraction observed is equal to the goal, the payoff value, y, should equal 50.
3. As the fraction observed becomes greater than the goal, the value slowly decreases below 50 to a minimum of 0.
4. As the fraction observed becomes less than the goal, the value slowly increases above 50 to a maximum of 100.
5. For any goal fraction, g, the increasing values moving from 50 to 100 will change at the same rate as the decreasing values from 50 to 0.
6. For any goal fraction, g, the complete range of values from 0 to 100 will be possible.

The policy is reflected by the following expressions. The developments of these expressions are given in Appendix C.

When $0 < g \leq .5$ and $0 \leq f \leq 2g$

$$y = 50 + \frac{50}{g^3}(g - f)^3$$

For the special case

$g = 0$ and $f \neq 0$, then $y = 0$
$g = 0$ and $f = 0$, then $y = 50$

When $0 < g \leq .5$ and $2g < f \leq 1$

$$y = 0$$

When $.5 < g < 1$ and $(2g - 1) \leq f \leq 1$

$$y = 50 + \frac{50}{(1 - g)^3}(g - f)^3$$

For the special cases

$g = 1$ and $f \neq 1$, then $y = 100$
$g = 1$ and $f = 1$, then $y = 50$

When $.5 < g < 1$ and $0 \leq f < (2g - 1)$

$$y = 100$$

Notice that a small power (3) was chosen to approximate this policy. Other odd powers could be used if appropriate.

Figure 11 displays the sketches for three values of g.

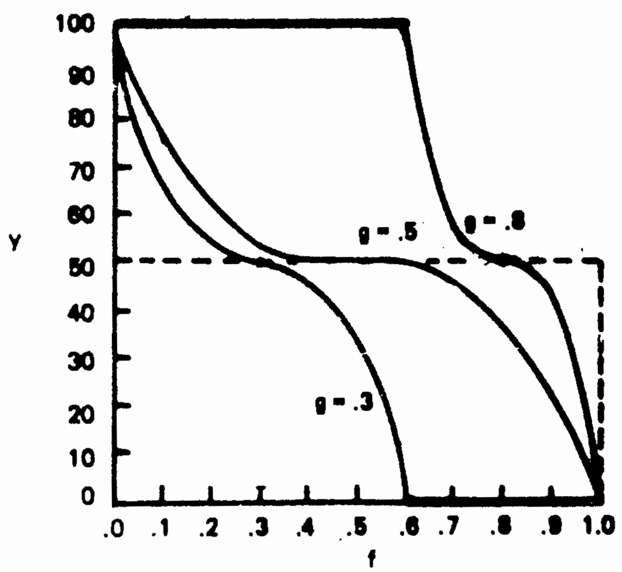

Figure 11. Y (payoff) = function at f(fraction fill) for g = .3, g = .5, g = .8.

Values of y are presented in Table 4 for various values of f and g.

Table 4 **Y(Payoff) = Function of g(goal) and f(fraction fill)**

		FRACTION FILLED										
		.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
	1.0	100	100	100	100	100	100	100	100	100	100	50
	.9	100	100	100	100	100	100	100	100	100	50	0
	.8	100	100	100	100	100	100	100	56	50	44	0
	.7	100	100	100	100	100	65	62	50	48	35	0
G	.6	100	100	100	71	56	51	50	49	44	29	0
O	.5	100	76	61	53	50	50	50	47	39	24	0
A	.4	100	71	56	51	50	49	44	29	0	0	0
L	.3	100	65	52	50	48	35	0	0	0	0	0
	.2	100	56	50	44	0	0	0	0	0	0	0
	.1	100	50	0	0	0	0	0	0	0	0	0
	0	50	0	0	0	0	0	0	0	0	0	0

In actual operation with this component it may be desirable to use a default option by imposing the condition y = 50 when g = 0 or g = 1. This results in an "on target" assumption for g = 0 or g = 1 for any values of f. If this approach is used for default, then it would be necessary to use a g value very close to zero to represent a goal of 0 and to use a g value very close to one to represent a goal of 1.

Example 4: Value to the Air Force as a Function of Several Components

This example involves weighting several components to express value in a way that controls the "contribution" or "relative weighting" of each component in the composite.

In this case the policy is described as follows:

1. The range of the composite value, y, is from 0 to 1000.

2. The "relative weighting" or "contribution" of a composite is determined by converting the range of each component into a specified fraction of the total composite indicated as follows.

Letting

x_i = value of variable i, i = 1, . . . , n

f_i = fraction of composite to be used by variable i

h_i = high value of variable i

l_i = low value of variable i

h_c = high value of composite

l_c = low value of composite

Then the model developed in Appendix D to express the previous policy is

$$y = l_c + \sum_{i=1}^{n} f_i \frac{(h_c - l_c)}{(h_i - l_i)} (x_i - l_i)$$

For example, let

$n = 2$

$h_1 = 100$ $h_2 = 50$ $h_c = 1000$

$l_1 = 0$ $l_2 = 0$ $l_c = 0$

$f_1 = .6$ $f_2 = .4$

$$y = l_c + f_1 \frac{(h_c - l_c)}{(h_1 - l_1)} (x_1 - l_1) + f_2 \frac{(h_c - l_c)}{(h_2 - l_2)} (x_2 - l_2)$$

$$y = 0 + .6 \frac{(1000 - 0)}{(100 - 0)} (x_1 - 0) + .4 \frac{(1000 - 0)}{(50 - 0)} (x_2 - 0)$$

$$y = 6x_1 + 8x_2$$

Figures 12 and 13 present several sketches of this function.

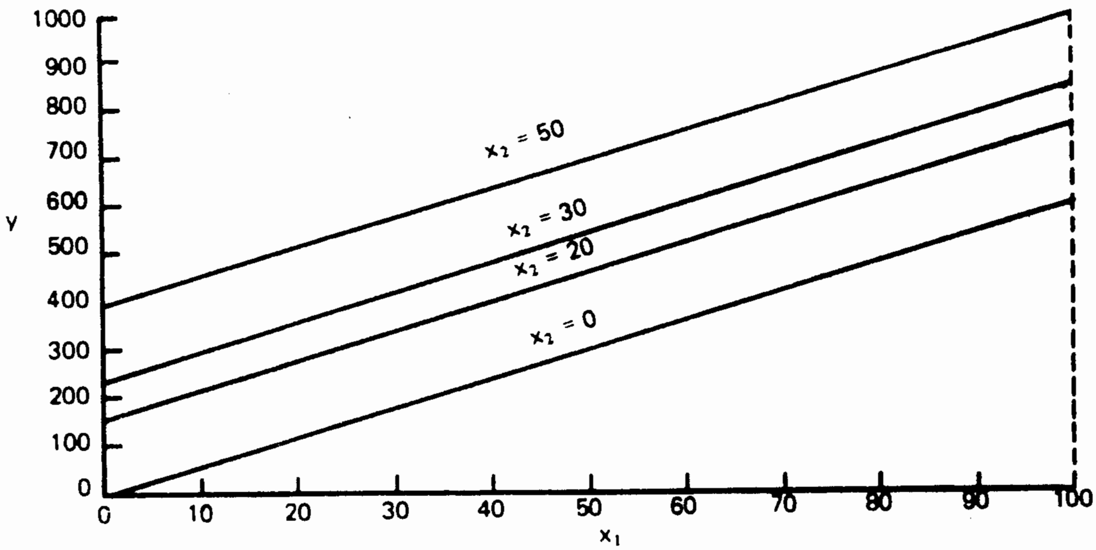

Figure 12. **Y (payoff) = function of x_1 at $x_2 = 0$, $x_2 = 20$, $x_2 = 30$, $x_2 = 50$.**

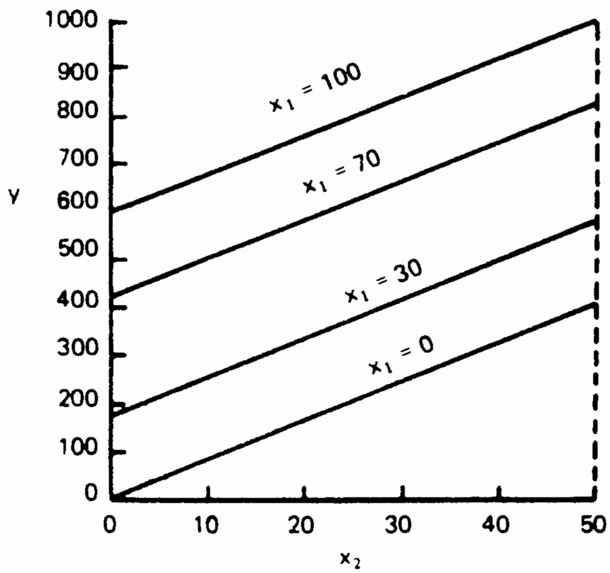

Figure 13. **Y (payoff) = function of x_2 at $x_1 = 0$, $x_1 = 30$, $x_1 = 70$, $x_1 = 100$.**