AFHRL-TR-74-58

AIR FORCE

HUMAN RESOURCES

THE ASSIGNMENT OF AIRMEN BY SOLVING THE TRANSPORTATION PROBLEM

By

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July 1974

Final Report for Period November 1973 – June 1974

LABORATORY

AIR FORCE SYSTEMS COMMAND
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NOTICE

This final report was submitted by Manpower and Personnel Systems Division, Air Force Human Resources Laboratory, Lackland Air Force Base, Texas 78236, under project 2077, with Hq Air Force Human Resources Laboratory (AFSC), Brooks Air Force Base, Texas 78235.

This technical report has been reviewed and is approved.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFHRL-TR-74-58	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) THE ASSIGNMENT OF AIRMEN BY SOLVING THE TRANSPORTATION PROBLEM		5. TYPE OF REPORT & PERIOD COVERED Final Nov 73 – June 74 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(*s*) Manuel Pina		8. CONTRACT OR GRANT NUMBER(*s*)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Manpower and Personnel Systems Division Air Force Human Resources Laboratory Lackland Air Force Base, Texas 78236		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 20770311
11. CONTROLLING OFFICE NAME AND ADDRESS HQ Air Force Human Resources Laboratory (AFSC) Brooks Air Force Base, Texas 78235		12. REPORT DATE July 1974 13. NUMBER OF PAGES 12
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

assignment	payoff value
fill	personnel
fit	TLS algorithm
maximization	Training Line Simulator (TLS)
optimization	transportation problem

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

To help the Air Force managers understand some of the many optimization techniques; two techniques, one which directly optimizes payoff values and one which indirectly optimizes payoff values, are discussed. In discussing these optimization techniques, the policy of fill and the policy of fit are discussed since they are an important part of both techniques. The techniques for solving the transportation problem will give the Air Force manager some insight into at least two assignment policies and two optimization techniques and will pose questions which must be considered in trying to optimize assignment of Basic Military Training graduates to their Air Force Specialty Code (job).

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PREFACE

This work was performed under Project 2077, Personnel and Manpower Management Systems Development; Task 207703, Computer-Based Models of Air Force Based Subsystems; Work Unit 20770311, Comparison of the Simplex Method, Transportation Method, Training Line Simulator (TLS), Assignment Algorithm (Ford-Fulkerson).

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I. INTRODUCTION

The projected FY 75 enlisted accession requirement of the Air Force is 78,000 individuals (annual Defense Department Report: 1975). Most of these accessions will be non-prior service enlistees. It is expected that about 55 percent of these non-prior service enlistees will enter the Air Force under the Guaranteed Training Enlistment Program (GTEP), the remainder will enter the Air Force with open enlistment status. The assignment of the latter group from basic military training to technical training or to a direct duty assignment is a formidable task for assignment personnel. There are various assignment policies available for tasks of this kind. They will be discussed in this report as "Fill Policy," "Fit Policy" or a combination of Fit/Fill. Two methods for implementing a Fit/Fill policy will be discussed.

II. FILL POLICY

A fill policy takes job vacancies as of a given time and maximally fills these vacancies from available manpower resources with only one constraint – that each individual assigned meets mandatory requirements for the job to which he is assigned. In other words, a fill policy optimizes the number of assignments without consideration of the best person-job match considerations within the set of job openings and available people.

III. FIT POLICY

Fit policy is to optimize the person-job match without regard to the number of jobs filled. One way to accomplish this optimized person-job match is to assign a payoff value (p_{ij}) greater than zero to a person who qualifies for a job. The magnitude of the p_{ij} is based on the degree of qualification. A p_{ij} of zero is assigned to all individuals who do not meet minimum qualifications for the job based on mandatory requirements. Mandatory and desirable prerequisites should enter into the computation p_{ij} values for qualified individuals. Maximizing fit can be achieved by maximizing the sum of the p_{ij}'s using this procedure. Except in special cases, the optimizing of fit does not mean the optimization of fill (example 1).

Example 1

Person \ Job	1	2	3
1	0	50	30
2	0	15	5
3	40	10	100

To maximize fit, only person 1 would have been assigned to job 2, person 3 to job 3, person 2 left unassigned, and job 1 unfilled. A total payoff of 150 would be achieved. If fill is maximized, the highest total payoff is 95 by assigning person 3 to job 1, person 1 to job 2, and person 2 to job 3. Fill could have also been maximized by assigning man 3 to job 1, person 2 to job 2, and person 1 to job 3. However, fit would not have been maximized since in this case, the total payoff would have been 85. Example 1 shows that optimizing fit does not necessarily optimize fill.

Example 2 shows that in some cases, fit and fill can be optimized at the same time. There are three ways in example 2 to achieve a maximum payoff of 140 and assign 3 persons to 3 jobs. But this is not always possible as shown in example 1.

Example 2

Person \ Job	1	2	3
1	10	20	30
2	40	50	0
3	60	70	80

Although the Air Force tries to optimize fill, fit is still considered, although not optimized; that is, the assignment of the best qualified person to jobs is made according to the prerequisites of the job, but always keeping fill at as high a level as possible. With congressionally imposed decreases on year-end manpower ceilings, it becomes more important to maximize the productivity of each airman. This will require greater attention to fit.

As mentioned earlier, the predicted payoff value of a person on a job can be used to define optimal fit in assigning a group of persons to jobs. Each person is given a potential payoff depending on how closely they meet the requirements for that job. Once every person has been given a payoff for each job, the persons are assigned in such a way that the sum of the payoffs is maximum. By maximizing the sum of the payoff of the assigned airmen, we optimize fit. For example, a payoff of zero would mean that this airman does not meet the mandatory requirements for this job. The greater the payoff, the more closely they meet mandatory and desirable requirements. The maximum payoff value a person could receive on one job in which they meet all the requirements could be different from the maximum payoff value they could receive on another job in which they meet all the requirements. This means that one job would have a greater payoff than another for such reasons as importance, priority, higher requirements, etc. For example, suppose job X and job Y are two jobs for which a person is being considered. If job X requires a very high aptitude airman and job Y does not, such a high aptitude airman could possibly be favored. If only two jobs and one person are involved, then the payoff on job X should be made greater than the payoff on job Y to increase the likelihood of the person being assigned to job X since the sum of the payoff values is maximized.

It is not true that fit maximization always assigns persons to jobs for which they are best qualified, but it is true that most persons tend to be assigned to the jobs for which they are best qualified. The overall assignments will be the best combination of persons to job assignments. Example 3 shows that optimizing fit does not necessarily assign the best person to each job.

Example 3

Person	Job 1	Job 2
1	10	130
2	20	150

To optimize the sum of the payoff values, person 1 is assigned to job 1 and person 2 to job 2 for a total payoff of 160. The assignment of the best person to each job is impossible in this case since person 2 is the best person for both jobs.

Technology is available to efficiently maximize the total payoff values of a payoff matrix. Development of reliable and realistic payoff values of persons for jobs is the key to success of this type policy.

IV. TRANSPORTATION PROBLEM

One way of combining fit and fill into a single policy is to view assignments of persons to jobs as the classical transportation problem described in linear programming and operations research textbooks. Under the fit policy the maximizing of the sum of payoff values can lead to a solution which maximizes fit but not necessarily fill. This was shown in example 1 when the payoff values were maximized and one person was left unassigned. In this case, when payoff values were maximized, the number of persons assigned was not the maximum number that could be assigned.

By adding the constraint to the problem of maximizing the sum of payoff values that the number of persons assigned will equal the number of jobs available, the problem becomes the transportation problem. The mathematical model for this is shown in equation (1):

$$\text{maximize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} \; x_{ij} \qquad (1)$$

subject to the restrictions

(1) $x_{ij} = 0 \text{ or } 1$

(2) $\sum_{j=1}^{n} x_{ij} = a_i$

(3) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (Supply = Demand)

(4) $\sum_{i=1}^{m} x_{ij} = b_j$

where p_{ij}, a_i and b_j are known parameters.

Jobs

		1	2	3	•	•	•	n	Supply
	1	p_{11}	p_{12}	p_{13}				p_{1n}	$a_1=1$
	2	p_{21}	p_{22}	p_{23}				p_{2n}	$a_2=1$
	3	p_{31}	p_{32}	p_{33}				p_{3n}	$a_3=1$
PERSONS	•								•
	•								•
	•								•
	m	p_{m1}	p_{m2}	p_{m3}				p_{mn}	$a_m=1$
Demand (Quota)		b_1	b_2	b_3	•	•	•	b_n	

meaning that $\sum_{j=1}^{n} b_j = m = \sum_{i=1}^{m} a_i$

Each a_i represents (supply) one person, therefore $a_i=1$ for all i. Each b_j represents (demand) the quota for that job, therefore $b_j \geqslant 1$.

The transportation problem can be solved by many different techniques, such as the Simplex Method, the Ford-Fulkerson Network Flow Algorithm, Dwyer's technique, and other methods (Hillier & Liberman, 1967).

In order to solve the transportation problem, it is necessary to have the same number of persons and jobs. If this is not the case, "dummy jobs" or "dummy persons" are created to make them equal. By convention, unqualified persons are assigned a payoff value of zero, but this does not prohibit the solution algorithm from assigning a person to a zero payoff job. Then as indicated above, the objective function is maximized within the constraints specified. In maximizing the objective function, persons will be assigned jobs where in some cases they are not qualified. These persons can be assigned to jobs for which they are not qualified as long as the objective function is maximized and the constraints met. This solution is the mathematical solution of the problem. However, the mathematical assignments are not the actual final assignments made since some of the assignments are not valid. The actual assignments are the mathematical assignments in which the persons are qualified. Assignments from the mathematical solution which are not qualified are reported as unassigned. Therefore, qualified assignments are a subset of the mathematical assignments and are the actual assignments which are made and reported.

V. TRAINING LINE SIMULATOR (TLS)

A second way of combining fit and fill into a single policy has been developed (Hatch, Nauta, Pierce, & Pina, 1973) and called the Training Line Simulator algorithm. The policy used by the TLS is termed "fill," but is actually a "fill then fit" policy. After the largest number of persons have been assigned, optimizing fill, further optimization is undertaken. Keeping the number of persons assigned equal to the fill optimization, persons are moved about to different jobs until the fit policy is maximized. Since the same number of persons must remain assigned, persons exchange jobs in order to improve fit. For example, suppose person 1 is assigned to job 1, person 2 is assigned to job 2, person 2 is more qualified for job 1 and they are equally qualified for job 2. Then by moving person 1 to job 2 and person 2 to job 1, fit is optimized. Note that either assignment optimizes fill in this case.

By making the payoff values equal to zero if a person is not eligible for a job and one when they are eligible for a job, all persons eligible for the same job are equally eligible. This problem is again the maximizing of the sum of payoff values subject to the mathematical assignment of all personnel and the meeting of all quotas. By using zero or one payoff values, no distinction is made as to whether a person is better assigned to one job or another.

A technique to use zero or one payoff values and still make a distinction between jobs is used by the TLS assignment algorithm. The TLS algorithm uses the Ford-Fulkerson procedure to maximize zero/one payoff problems. The TLS algorithm goes through a series of optimizations in order to avoid the use of payoff values directly. Each step is a solution to a transportation problem and optimizes fill since the maximization of the sum of the payoff values is equal to the maximum number of persons that can be assigned. This is true since the number of persons assigned is equal to the sum of the payoff values.

Input to the TLS assignment algorithm is BMTS graduates with all information needed to determine their eligibility for a job (AFSC) plus the mandatory prerequisites, the desirable prerequisites and quotas for each job (AFSC). The mandatory prerequisites are requirements which a BMT graduate must meet before he can be considered eligible for a job. Desirable prerequisites are prerequisites which are not necessary and are considered only after the person has met the mandatory prerequisites for the job. The mandatory prerequisites are the minimum requirements which an airman must meet to be eligible for an AFSC while desirable prerequisites are requirements which are considered valuable for an airman to have to perform his job in this particular AFSC. A person can be assigned to an AFSC even if they do not meet any desirable prerequisites, but cannot be assigned to an AFSC if they do not meet all of the mandatory prerequisites.

Within the TLS model, the desirable prerequisites for each AFSC are arranged into levels of desirability. At each level, one or more of the desirable prerequisites are grouped to define the desirable requirements for the job at that particular level. From 1 to 25 levels may be used to describe the ranking which is desired for the assignment of eligible airmen to each AFSC. For example, for a particular job, the level 1 requirement could be an AQE of 95, a high school graduate, and a special test score of 90. The level 2 requirement could be an AQE of 90, a high school graduate, and a special score of 85. Each level after level 1 would have less stringent requirements.

The first step in the optimization process of the TLS algorithm is to develop a 0, 1 eligibility matrix. The matrix will be a matrix of persons versus jobs and will contain a zero or a one in each element. A one will be generated for each job for which a person meets the mandatory prerequisites and a zero if the person does not qualify for this particular job as far as the mandatory prerequisites. Each job has a quota which is equal to or greater than one (example 4).

Example 4

Person	Job 1	Job 2	Job 3	•	•	•	Job n
1	0	1	1				0
2	1	0	0				0
3	1	1	1				1
•							
•							
•							
m	1	0	1				0

Person 1 does not meet the mandatory requirements for job 1, but does meet them for job 2 and job 3. Once generated, the eligibility matrix is optimized and will assign the maximum number of airmen that is possible. That is, the transportation problem (equation 2) is solved:

$$\text{maximize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\; p_{ij} \qquad (2)$$

subject to the restrictions

(1) $x_{ij} = 0 \text{ or } 1$

(2) $\sum_{j=1}^{n} x_{ij} = a_i$

(3) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

(4) $\sum_{i=1}^{m} x_{ij} = b_j$

where

$a_i = 1$ (1 person)

$b_j = 1$ (1 job)

$p_{ij} = 0$ payoff if person is not eligible for job

$= 1$ payoff if person is eligible for job

Maximizing this transportation problem optimizes fill, the number of persons assigned is maximized.

After the optimization of the eligibility matrix, a 0, 1 matrix is generated for each level of desirability used. A zero being generated if the person does not meet the desirable prerequisites or the mandatory prerequisites and a one if the person meets the desirable and mandatory prerequisites.

Each matrix (starting with level 1) is then optimized. That is, for level 1, the following transportation problem (equation 3) is solved.

$$\text{maximize } Z(1) = \sum_{i=1}^{n} \sum_{j=1}^{n} x(1)_{ij} p_{ij} \qquad (3)$$

subject to the restrictions

(1) $x_{ij} = 0 \text{ or } 1$

(2) $\sum_{j=1}^{n} x_{ij} = a_i$

(3) $\sum_{i=1}^{n} a_i = \sum_{j=1}^{n} b_j$ (Supply = Demand)

(4) $\sum_{i=1}^{n} x_{ij} = b_j$

(5) Z must remain maximum

where $p_{ij} = 0 \text{ or } 1$

For level 2, the following transportation problem (equation 4) is solved.

$$\text{maximize } Z(2) = \sum_{i=1}^{n} \sum_{j=1}^{n} x(2)_{ij} p_{ij} \qquad (4)$$

subject to the restrictions

(1) $x_{ij} = 0 \text{ or } 1$

(2) $\sum_{j=1}^{n} x_{ij} = a_i$

(3) $\sum_{i=1}^{n} a_i = \sum_{j=1}^{n} b_j$ (Supply = Demand)

(4) $\sum_{i=1}^{n} x_{ij} = b_j$

(5) Z and Z(1) must remain maximum

where $p_{ij} = 0 \text{ or } 1$

Continue to optimize transportation problems until every level has been optimized. The optimization accomplished in each case is the maximization of fill constrained by previous solutions. Once all levels have been maximized, "max fit," the optimization of assignment by the desirable prerequisites, has also been accomplished.

The number of persons assigned by the first optimization, optimization of the mandatory eligibility matrix, will remain the same. Starting with the highest level (level 1), assignments are rearranged so that the maximum number of persons qualify at this level subject to the constraint that the number already assigned remain the same. Then subject to the optimal solutions found in higher desirability levels and keeping the number already assigned the same, the assignments are again rearranged to obtain the maximum number of assignments within this desirability level. The final results will be: (1) the optimization of quota accommodation (fill) and (2) the optimization within the constraints imposed by (1) of desirable prerequisites (fit) (Hatch, Nauta, Pierce, & Pina, 1973).